EFFECTIVENESS OF CONCRETE AND COMPUTER SIMULATED MANIPULATIVES ON ELEMENTARY STUDENTS' LEARNING SKILLS AND CONCEPTS IN EXPERIMENTAL PROBABILITY

By

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This dissertation is dedicated to my mother,

Laura Ann Roberts,

whose love and belief I was never without

and

to my husband,

Reginald Eugene Taylor,

my mate and friend.

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This study was designed to investigate the impact of using concrete manipulatives and computer-simulated manipulatives on elementary students' learning skills and concepts in experimental probability. Of primary interest to the researcher was students' ability to predict outcomes of simple experiments. A secondary interest was the incidental fraction learning that might occur as students engage in the experimental probability experiences.

The research sample consisted of 83 fifth-grade students. There were four treatment groups. Treatment Group I students received computer instruction. Treatment Group II students received manipulative guided instruction. Treatment Group III students received both computer instruction and manipulative guided instruction. Treatment Group IV served as a control group and received traditional instruction. The researcher used an analysis of covariance (ANCOVA). After controlling for initial

differences, the researcher concluded that students experiencing the computer instruction only significantly outperformed students experiencing a traditional instructional environment regarding experimental probability learning skills and concepts.

CHAPTER 1
DESCRIPTION OF THE STUDY

Introduction

While the curriculum in the United States continues to focus on basic skills well past the fourth grade, classrooms in Japan and Germany emphasize more advanced concepts--including algebra, geometry, and probability (Riley, 1998). The aim of the United States is to be first in science and mathematics, teachers may need to introduce children to more advanced topics at earlier grade levels. *Principles and Standards for School Mathematics*, published by the National Council of Teachers of Mathematics (NCTM, 2000), recognizes probability as an advanced topic that should be learned by students in the K-12 curriculum. In the *Principles and Standards*, emphasis is placed on introducing the same topics and concepts throughout the Pre K-12 curriculum. Young children need to explore the process of probability. The study of probability in the early grades provides a stronger foundation for high school students (NCTM, 2000). Bruner (1960) stated in his discussion of the spiral curriculum,

> If the understanding of number, measure, and probability is judged crucial in the pursuit of science, then instruction in these subjects should begin as intellectually honestly and as early as possible in a manner consistent with the child's forms of thought. Let the topics be developed and redeveloped in later grades. (pp. 53-54)

He also believed,

> If one respects the ways of thought of the growing child, if one is courteous enough to translate material into his logical forms and

> challenging enough to tempt him to advance, then it is possible to introduce him at an early age to the ideas and styles that in later life make an educated man. (p. 52)

The foregoing reasons seem to point to the teaching of experimental probability concepts beginning in the elementary grades.

<u>Probability</u>

"As with other beautiful and useful areas of mathematics, probability has in practice only a limited place in even secondary school instruction" (Moore, 1990, p. 119). The development of students' mathematical reasoning through the study of probability is essential in daily life. Probability represents real-life mathematics. Probability also connects many areas of mathematics, particularly numbers and geometry (NCTM, 1989). "Research in medicine and the social sciences can often be understood only through statistical methods that have grown out of probability theory" (Huff, 1959, p. 11). Moore (1990) stated,

> Probability is the branch of mathematics that describes randomness. The conflict between probability theory and students' view of the world is due at least in part to students' limited contact with randomness. We must therefore prepare the way for the study of chance by providing experience with random behavior early in the mathematics curriculum. (p. 98)

An understanding of probability theory is essential to understand such things as politics, weather reports, genetics, state lotteries, sports, and insurance policies. As shown Table 1, the list of questions presented in the different areas shows the need for experimental probability. These questions require considerations of probabilities and what they mean (Huff, 1959).

Table 1

Real Life Mathematics Using Experimental Probability

Example	Description
Politics	A random survey of voters show that 381 out of 952 are planning to vote for Candidate McCain in the primary election. What is the probability that a randomly selected voter will vote for Candidate McCain?
Weather Reports	The probability of rain today is 70%. What are the odds in favor of rain? What are the odds against rain?
Genetics	In genetics, we can use probability to estimate the likelihood for brown-eyed parents to produce a blue-eyed child. The gene for brown eyes in humans is dominant over the gene for blue eyes. If each parent has a dominant brown and a recessive blue gene, what is the chance that their child will have blue eyes?
State Lotteries	A lottery consists of choosing 6 numbers, the first 5 numbers being one of the digits 0-9 and the sixth number being from 0-39. How many possible sets of lottery numbers are available for selection? If the winning numbers are randomly selected, what is the probability of winning with a single ticket?
Sports	Softball statistics (NCTM, 1989, p. 111) Home runs 9 Triples 2 Doubles 16 Singles 24 Walks 11 Outs 38 Total 100 Above is the record of a player's last 100 times at bat during the softball season. What is the probability the player will get a home run? What is the probability that the player will get a hit?
Insurance Policies	Insurance companies use the principle of probability to determine risk groups. One way of figuring out what insurance you need is by looking at the probability of the event and the financial loss it would cause. What is the probability of men between 18-25 years of age being involved in a car accident? Determine if the cost of each occurrence is high or low.

The inclusion of activities dealing with experimental probability in the elementary school enhances children's problem-solving skills and provides variety and challenges for children in a mathematics program (Kennedy & Tipps, 1994). Current and past recommendations for the mathematics curriculum identify experimental probability as one of several critical basic skill areas that should occupy a more prominent place in the school curricula than in the past (National Council for Supervisor of Mathematics (NCSM), 1989; Mathematical Sciences Education Board [MSEB], 1990; Willoughby, 1990; NCTM, 2000).

From a historical perspective, members of the Cambridge Conference on School Mathematics (1963) also acknowledged the role probability and statistics played in our society. The Cambridge Conference was an informal discussion of the condition of the mathematics curriculum in the United States at the elementary and secondary level. The members of the conference addressed revisions to the mathematics curriculum. They recommended that probability and statistics not only be included as part of the modern mathematics of that day; but also these were recommendations that they considered for 1990 and 2000. Other researchers have also suggested that elements of statistics and probability be introduced in the secondary school curriculum and possibly at the elementary level as part of the basic literacy in mathematics that all citizens in society should have (Schaeffer, 1984; Swift, 1982).

Therefore, changes are being made today to introduce probability into the elementary school curriculum (NCTM, 2000). Experience with probability can contribute to students' conceptual knowledge of working with data and chance (Pugalee, 1999). This experience involves two types of probability--theoretical and experimental.

There may be a need for students to be exposed to more theoretical models involving probability. Theoretical models organize the possible outcomes of a simple experiment. Some examples of theoretical models may include making charts, tree diagrams, a list, or using simple counting procedures. For example, when asked to determine how many times an even number will appear on a die rolled 20 times, students can list the ways of getting an even number on a die (2, 4, 6) and may conclude that one should expect an even number one-half of the time when a die is rolled. Then, students can roll the die 20 times, record their actual results and make conclusions based on their experiment. Experimental models are then the actual results of an experiment or trial. Another example that involves experimental modeling is the following:

> If you are making a batch of 6 cookies from a mix into which you randomly drop 10 chocolate chips, what is the probability that you will get a cookie with at least 3 chips? Students can simulate which cookies get chips by rolling a die 10 times. Each roll of the die determines which cookie gets a chip. (NCTM, 1989, pp. 110-111)

The researcher's interest was with experimental probability and not on theoretical probability but used the term probability throughout the study, since there is no way to teach experimental without theoretical.

A secondary interest with experimental probability involves incidental fraction learning. Incidental learning is unintentional or unplanned learning that results from other activities. Proponents of incidental learning believe that effective learning can take place this way. For example, the incidental learning theory suggests that children learn mathematics better if it is not methodically taught to them (Clements & Battista, 1992). In addition, Brownell (1935) suggested that incidental learning could help counteract the practice of teaching arithmetic as an isolated subject. Studying and solving probability

problems give students many opportunities for practicing and reinforcing previously learned concepts of basic mathematics skills (Horak & Horak, 1983; NCTM, 2000) that, according to the Allen, Carlson, and Zelenak (1996), are inadequate. "For example, fraction concepts play a critical role in the study of probability . . . as well as whole number operations and the relationships among fractions, decimals, and percents" (NCTM, 1989, p. 111).

There are also opponents of incidental learning. These opponents believe that although the study of probability may provide incidental learning for fractions, it may not be enough to develop students' ability and concepts of fractions. They also suggest that incidental learning does not provide an organization for the development of meaningful concepts and intelligent skills in which the development of authentic mathematic ability is possible (Brownell, 1935).

Manipulatives

The use of different modes of representation can promote meaningful learning, retention, and transfer of mathematical concepts (Lesh, 1979). One example includes the use of manipulatives. Manipulatives may be physical objects (e.g., base ten blocks, algebra tiles, pattern blocks, etc.) that can be touched, turned, rearranged, and collected. Manipulatives may be real objects that have social application in everyday situations, or they may be objects that are used to represent an idea. Manipulatives enhance mathematics achievement for students at different grade levels (Suydam, 1984). Various types of manipulatives used for teaching and learning mathematics are presented as follows: tangrams, cuisenaire rods, geoboards, color tiles, pattern blocks, coins, color spinners, number spinners, snap cubes, base ten blocks, dice, fraction strips, dominoes,

clock dials, color counters, and attribute blocks. Examples of the mathematics content for some of the manipulatives are given:

1. Geoboards can be used for activities involving geometric shapes, symmetry, angles and line segments, and perimeter and fractions in a problem-solving context.

2. Tangrams can be used for creating and comparing size and shape, measurement, properties of polygons, and transformations as well as developing spatial visualization.

3. Color tiles can be used to teach topics such as fractions, percents, and ratios; probability, sampling, graphing, and statistics; and measurement.

Students' knowledge is strongest when they connect real-world situations, manipulatives, pictures, and spoken and written symbols (Lesh, 1990). Manipulatives help students construct meaningful ideas and learn mathematics more easily (Burns, 1996; Clements & McMillan, 1996).

> Support for the use of manipulatives come from Piagetian theory, in which cognitive development is described as moving from concrete to abstract, through a series of developmental stages that are roughly age-related. A concrete-operational child cannot handle abstract concepts before arriving at the appropriate stage. However, with manipulatives, it is possible for such a student to take the first steps towards exploring the concepts; manipulatives are concrete introductions to abstract ideas. (Perl, 1990, p. 20)

The use of manipulative materials appears to be of definite importance in how well children understand and achieve in mathematics in different content areas.

<u>Technology</u>

Technology takes a special place in the child-driven learning environment as a powerful tool for children's learning by doing (Strommen & Lincoln, 1992). The NCTM (2000) technology principle indicated that mathematics instructional programs should use

technology to help all students understand mathematics and prepare students to use mathematics in a growing technological world. The use of computer technology is an integral part of the vision of the future of teaching and learning mathematics. One of the position statements of NCTM (1998) is as follows:

> The appropriate use of instructional technology tools is central to the learning and teaching of mathematics and to the assessment of mathematics learning at all levels. Technology has changed the ways in which mathematics is used and has led to the creation of both new and expanded fields of mathematical study. Thus, technology is the driving change in the content of mathematics programs, in methods for mathematics instruction, and in the ways that mathematics is learned and assessed. [On-line]

Inclusion of technology in classroom projects enhances the authenticity of the task (Greening, 1998).

Technological aids allow greater realism in the classroom (MSEB, 1990). Progression in technology has increased the boundaries of mathematics and emphasized the importance of the integration of technology in the mathematics curriculum. At every grade level, there are ways in which students can experience mathematics with technology that provide deeper and more substantial understanding (NCTM, 2000).

Children's traditional classroom tools--pencil, notebooks, and texts--are still vital but inadequate for children to adequately solve problems, completely modify ideas, and thoroughly extend their learning experience. "Rather than simply listening to teachers talk, watching the teacher write symbolic procedures on the board, and doing pencil and paper practice, children should learn through meaningful hands-on activities with manipulative materials, pictures, and technology" (Weibe, 1988, p. 66). "Computers can provide an important link in the chain, a connection between the concrete manipulatives and the abstract, symbolic, paper and pencil representation of the mathematical idea"

(Perl, 1990, p. 21). From a Vygotskian perspective, "appropriately implemented, technology offers tools to the classroom that may promote high level thinking skills and support concept development" (Harvey & Charnitski, 1998). Clements (1998) also believed that the computer can provide practice on arithmetic processes and foster deeper conceptual thinking.

Computer simulations can help students develop insight and confront misconceptions about probabilistic concepts. "With the computer and an LCD [Liquid Crystal Diode] panel such as the PC [Personal Computer] viewer, this process becomes easier and more powerful" (Perl, 1990, p. 22).

> Probability concepts and their meaning depend not only on the level of theory and on their representation but also the means of working with them. Tools to represent knowledge or to deal with knowledge have a sizeable impact on subjects' individual formation of this knowledge. Clearly, the advent of computers marks a huge change . . . the computer will radically change available means of working on problems and means of representation. (Kapadia & Borovcnik, 1991, p. 21)

An advantage of simulation by computer as compared to physical random generators is the possibility for larger numbers of repetitions and extensive exploration of assumptions (Kapadia & Borovcnik, 1991). Although this is true, students in the elementary grades need to work with random events first generated through physical simulation and then followed by simulations done electronically using available software. This would lead to deeper understandings as students reach middle and high school (NCTM, 2000). Moore (1990) stated,

> The first step toward mathematical probability takes place in the context of data from chance devices in the early grades. Computer simulation is very helpful in providing the large number of trials required if observed relative frequencies are to be reliably close to probabilities. (p. 120)

Moore also stated, "Simulation, first physical and then using software can demonstrate the essential concepts of probability" (p. 126).

Constructivism

The constructivist perspective on learning suggests that knowledge is not received passively; it has to be built up (Maher, 1991). Constructivists generally agree on the following basic tenets (Noddings, 1990):

> 1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
>
> 2. There exist cognitive structures that are activated in the processes of construction. These structures account for the construction; that is they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.
>
> 3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
>
> 4. Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.
>   a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
>   b. Pedagogical constructivism suggest methods of teaching consonant with cognitive constructivism (p. 10).

Essentially, students construct knowledge from their own experiences--personal and academic. One goal from a constructivist perspective is for students to develop mathematic structures that are more complex, abstract, and powerful than what students currently possess (Clements & Battista, 1990; Cobb, 1988). Teachers who adhere to a constructivist perspective emphasize understanding and building on

students' thinking. A comparison of things that teachers with a constructivist perspective do differently compared to a traditional curriculum is listed in Table 2.

Table 2

Differences Between Two Teaching Styles

Traditional	Constructivist
Emphasis is on basic skills	Emphasis is on big concepts
Teachers follow fixed curriculum guidelines.	Teachers allow student questions to guide the curriculum.
Activities rely heavily on texts and workbooks	Activities rely on primary sources of data and manipulatives
Teachers give students information; students are viewed as black slates.	Teachers view students as thinkers with emerging theories about the world.
Students mostly work alone. Teachers look for correct answers to assess learning.	Students primarily work in groups. Teachers seek students' points of view to check for understanding.
Assessment is viewed as separate from teaching and occurs mostly through testing.	Assessment is interwoven with teaching and occurs through observation and student exhibits and portfolios.

(Anderson, 1996)

Concrete manipulatives and computers can be a powerful combination in the mathematics curriculum (Perl, 1990). There has been a growing need to investigate how to integrate the use of computers with manipulative materials to facilitate mathematics instruction and to meet the needs of different learners (Berlin & White, 1986). Aligning computer usage and concrete manipulatives with the constructivist perspective may help

to improve students' performance in mathematics. For example, manipulative materials appeal to several senses and are characterized by a physical involvement of pupils in an active learning situation (Reys, 1971).

In addition, computers also serve as a catalyst for social interaction with other students--allowing for increased communication (Clements, 1998). Social interaction constitutes a crucial source of opportunities to learn mathematics (Piaget, 1970). Involving children in the process of doing mathematics and using concrete materials relates to their development where children are actively thinking about the mathematics. While children work with objects and discuss what they are doing, they begin to recognize a sense of relationship among mathematics concepts (Suydam, 1984) that support their learning.

Purpose and Objectives of the Study

The purpose of this study was to examine the effectiveness of concrete manipulatives and computer-simulated manipulatives on elementary students' experimental probability learning skills and concepts. The objectives of the study were as follows:

1. To determine the effectiveness of computer-simulated manipulatives on elementary students' learning skills and concepts in experimental probability,

2. To determine the effectiveness of concrete manipulatives on elementary students' learning skills and concepts in experimental probability,

3. To determine the effectiveness of both concrete manipulatives and computer-simulated manipulatives on elementary students' learning skills and concepts in experimental probability, and

4. To determine if incidental fraction learning occurs as students engage in the experimental probability experiences.

Rationale for the Study

The most important use of studying probability is to help us make decisions as we go through life (Newman, Obremski, & Schaeffer, 1987). For example, in issues of fairness, students may pose a question based on claims of a commercial product, such as which brand of batteries last longer than another (NCTM, 2000).

Benefits of Probability Knowledge in Early and Later Grades

The committee for the *Goal for School Mathematics*, the report of the 1963 Cambridge Conference, recommended introducing basic ideas of probability very early in the school program. The study of probability allows a learner to make sense of experiences involving chance.

If Studied in Early Grades, Teachers Need Information

As previously mentioned in the Introduction, Bruner (1960) believed that by constantly reexamining material taught in elementary and secondary schools for its fundamental character, one is able to narrow the gap between *advanced* knowledge and *elementary* knowledge. Bruner said that if you wish to teach calculus in the eighth grade, then begin in the first grade by teaching the kinds of ideas and skills necessary for the mastery of calculus in later years. This is one of the changes made in the *Principles and Standards*--to include the same concepts throughout all grade levels, teaching necessary skills appropriate at that grade level, and continuing at each grade level. Bruner believed concepts should be revisited at increasing levels of complexity as students move through the curriculum rather than encountering a topic only once (Schunk, 1991). This would also apply to teaching probability. If students are to understand probability at a deeper

level in high school and college, then the skills necessary for its mastery must begin in the elementary grades (NCTM, 2000). Therefore, it is important that evidence be available to help teachers develop appropriate topics in the elementary school curriculum. The kind of reasoning used in probability is not always intuitive, and so it may not be developed in young children if it is not included in the curriculum (NCTM, 2000).

Lack of Research in Early Grades

In the past, the teaching of probability reasoning, a common and important feature of modern science, was rarely developed in our educational system before college (Garfield & Ahlgren, 1988; Bruner, 1960). Currently, probability and statistics are often included in the secondary school curriculum only as a short unit inside a course (Shaughnessy, 1992, 1993). Recommendations concerning school curricula suggest that statistics and probability be studied as early as elementary school (MSEB, 1990; NCTM, 2000). However, much of the existing research on probability is on the conception of beginning college students or secondary school students (Shaughnessy, 1993). Very little research in the development of children's understanding of probability concepts has been done with young children (Watson, Collis, & Moritz, 1997). The learning of probability in early grades will provide students with a stronger foundation for further study of statistics and probability in high school.

Significance of the Study

Little research has been conducted about how to teach probability effectively in the early grades. Much of the research done on probabilistic intuitions of students from elementary school through college level has been conducted in other countries such as

Europe and Germany (Garfield & Ahlgren, 1988). If students in the United States are to be first in mathematics and science, there must be adequate research with a population of U.S. students to develop a better understanding of the difficulties these students encounter in understanding probability and statistics. It is not possible to learn much about learners unless we learn about learners' learning specific mathematics (Papert, 1980). This study provides information about children's learning of probability.

Many middle and high school students have difficulty understanding how to report a probability. Inadequacies in prerequisite mathematics skills and abstract reasoning are part of the problem (Garfield & Ahlgren, 1988). These difficulties may be due to little or no curriculum instruction for probability given at the elementary school level. The challenge is to relate to children and engage them in learning experiences in which they construct their own understanding of probability concepts.

There is a scarcity of literature in terms of teaching and learning probability from a constructivist perspective at the elementary school level. Recent research efforts emphasize constructivist approaches to teaching and learning mathematics (Thornton & Wilson, 1993). One implication for a constructivist theory of knowledge, according to Confrey (1990), is that students are continually constructing understanding of their experiences. In addition, this is the first research study conducted using the software developed by Drier (2000) in whole class instruction. Previous researchers (Clements & McMillan, 1991; Jiang, 1993) have concluded that computer-simulated manipulatives enhance students' mathematics learning ability. However, most of those studies involved older children. This researcher used computer-simulated manipulatives at the elementary level to teach probability. Because children's thinking processes are much less

developed, there is doubt about whether computer-simulated manipulatives are similarly effective for young children. Usually when probability is taught, it is done using concrete manipulatives. Because much of the research on probability learning skills and concepts has focused on university-level students, additional research is needed to create a framework from which to understand elementary students' probability learning better.

Organization of the Study

This chapter included the purpose of the study, its rationale, and its significance to the field of mathematics education. The review of relevant literature presented in Chapter 2 includes the theoretical framework, studies of probability, studies involving concrete manipulatives, and studies involving technology with a focus on computer-simulated manipulatives. Reported in Chapter 3 are the design and methodology of the study. Chapter 4 contains the results of the analysis and the limitations of the study. A summary of the results, implications, and recommendations for future research is presented in Chapter 5.

CHAPTER 2
LITERATURE REVIEW

Overview

In this chapter, the researcher presents a review of the relevant literature. The researcher focuses on studies pertaining to the following areas: constructivist theory, concrete manipulatives, studies involving technology with a focus on computer-simulated manipulatives, and studies of probability.

Theoretical Framework

Constructivism

A theory of instruction, which must be at the heart of educational psychology, is principally concerned with how to arrange environments to optimize learning and transfer or retrievability of information according to various criteria (Bruner, 1960). Learning is a process of knowledge construction, dependent on students' prior knowledge, and attuned to the contexts in which it is situated (Hausfathers, 1996). This definition is what the constructivist believes. Constructivism focuses our attention on how students learn. Constructivism implies that much learning originates from inside the child (Kamii & Ewing, 1996). The students construct their own understanding of each mathematical concept. Constructivism appears to be a powerful source for an alternative to direct instruction (Davis, Maher, & Noddings, 1990). Mathematical learning is an interactive and constructive activity (Cobb, 1988) that Constructivists in mathematics education

argue that cognitive constructivism implies pedagogical constructivism; that is, acceptance of constructivist premises about knowledge and knower implies a way of teaching that acknowledges learners as active knowers (Noddings, 1990). Direct instruction does not suit children's thinking because it is based quite often on a direct-teaching and practice model (Broody & Ginsburg, 1990). With direct instruction one finds a relatively familiar sequence of events: telling, showing, and doing approach (Confrey, 1990). It moves quickly, often overlooking students' development, preventing assimilation of higher cognitive skills (Broody & Ginsburg, 1990; Confrey, 1990).

We must help students develop skills that will enable them to be more productive when faced with real world situations. When students do not have the skills to solve everyday problems, they have difficulty finding a solution. Owen and Lamb (1996) gave the following example:

> John and Terry worked at a fruit stand during the summer. The two ran the register, which has a scale to weigh the fruit. One day the fruit stand lost electricity. The customers wanted to buy bananas, apples, and tangerines. John and Terry were not sure what to do without any electricity. They suggested waiting until the owner returned. Terry saw an old scale in the corner of the store. They decided to use the scale and figure out the cost of the fruit using paper and pencil. John and Terry tried to remember formulas taught to them in school earlier that year but were stumped. The bananas cost 39c per pound and the customer had 3-1/2 pounds of bananas. One boy suggested that when you have fruit problems, you divide the money by the weight of the fruit. They did this, came up with 11c, and thought that was incorrect because of the low price. They continued in this manner until they finally thought they found the right answer. John and Terry's teachers used teaching techniques that only provided them with minimal skills. Faced with a real life problem, John and Terry could not rely on skills taught to them by their teachers.

Constructivist Classrooms

Students learn through a social process in a culture or a classroom that involves discovery, invention, negotiation, sharing, and evaluation (Clements & Battista, 1990) and that actively constructs their learning. Social interactions constitute a crucial source of opportunities to learn mathematics that in the process of constructing mathematical knowledge involve cognitive conflict, reflection, and active cognitive reorganization (Piaget, 1970). Vygotsky (1978) had this to say about students' social interaction:

> Any function in the child's cultural development appears twice or on two planes. First, it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category . . . social relations or relations, among people genetically underlie all higher cognitive function and their relationships. (p. 57)

Vygotsky also stated,

> An essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of internal development processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. (p. 90)

To promote meaningful learning, teachers must know how to tailor instruction so that it meshes with children's thinking (Brownell, 1935; Dewey, 1963; NCTM, 1989; Piaget, 1970). When children have to give up their own thinking to obey the rules of the teacher (Kamii & Ewing, 1996), they are forced to compromise their own thinking. Children compromise their inventive, problem-solving methods for mechanical applications of arithmetic skills (Carpenter, 1985; Resnick & Omanson, 1987). Eventually, students may be interested only in getting the correct solution. In a constructivist classroom, emphasis is not on wrong or right answers; students construct

meaning from preexisting knowledge within the student. Completing objectives or preparing for tests is not the focus in the constructivist classroom. These do not sufficiently determine if learning occurred or if there are any changes in a student's conceptual understanding.

Schulte (1996) described a curriculum unit on weathering and erosion taught in a constructivist classroom. Students make predictions using hands-on exploration activities with unexpected outcomes that arouse curiosity. Terms have not yet been introduced. Students discuss the results of the activities with group members. The teacher poses questions looking for misconceptions, and activities are sequenced to challenge students. Once students begin to make connections between activities and understand the concept, the vocabulary is introduced. Questions focus on application and are used to look for student understanding. Alternative forms of assessment are used. For example, the students are to make a book on erosion including pictures, drawings, and text using landforms affected by erosion. Assessment may also focus on issues in society. For example, students may be asked to research methods that people are using to stop erosion in certain areas and design a beachfront home to build, considering the concept of erosion. Schulte believed that learning depends on the shared experiences of students, peers, and the teacher. He also felt that collaboration with others is important and that cooperative learning is a major teaching method used in the constructivist classroom.

Using the constructivist premise would imply there are many roads to most solutions or instructional endpoints (Noddings, 1990). Rote responses do not represent students' understanding of concepts. Emphasis on instruction forces us to probe deeper into students' learning and to ask the following questions (Noddings, 1990):

> 1. How firm a grasp do students have on the content?
>
> 2. What can students do with the content?
>
> 3. What misconceptions do students entertain?
>
> 4. Even if wrong answers are visible, are students constructing their learning in a way that is mathematically recognizable? (p. 14)

These address only a few questions at the heart of constructivist learning and teaching. Most curriculums pay little or no attention to the developmental abilities of children. Consequently, children do not learn much of what is taught to them (Brooks, 1987).

In a constructivist classroom there is consensus that the following things are noticeable (Anderson, 1996; Glatthorn, 1994; Schulte, 1996):

1. Teacher asks open-ended questions and allows wait time for responses.

2. Higher-level thinking is encouraged.

3. Students are engaged in experiences that challenge hypotheses and encourage discussion.

4. Classes use raw data, primary sources, manipulatives, physical and interactive materials.

5. Constructivism emphasizes the importance of the knowledge, beliefs, and skills an individual brings to the experience of learning; recognizes the construction of new understanding as a combination of prior learning, new information and readiness to learn.

In order for students to establish a strong foundation in mathematics, teachers need to provide experiences to students that involve forming patterns and relations, which are essential in mathematics (Midkiff & Thomasson, 1993). Caine and Caine (1991) posited the following:

> Children live with parallel lines long before they ever encounter school. By the time parallel lines are discussed in geometry, the average student has seen thousands of examples in fences, windows, mechanical toys, pictures, and so on. Instead of referring to the parallel lines students and teachers have already experienced, most teachers will draw parallel lines on the blackboard and supply a definition. Students will dutifully copy this "new" information into a notebook to be studied and remembered for test. Parallel lines suddenly become a new abstract piece of information stored in the brain as a separate fact. No effort has been made to access the rich connections already in the brain that can provide the learner with an instant "Aha!" sense of what the parallel lines they have already encountered mean in real life, what can be done with them, and how they exist than as a mathematical abstraction. (p. 4)

From a constructivist perspective, mathematical learning is not a process of internalizing carefully packaged knowledge but is instead a matter of reorganizing activity, where activity is interpreted broadly to include conceptual activity or thought (Koehler & Grouws, 1992).

Constructivism-Grounded Research

The constructivist teaching experiment is a technique designed to investigate children's mathematical knowledge and learning experiences (Cobb & Steffe, 1983; Cobb, Wood, & Yackel, 1992; Hunting, 1983). Several studies in mathematics education, inspired by the constructivist perspective, provided some information on children's thinking in mathematics (Bednarz & Janvier, 1988; Carpenter, Moser, & Rombert, 1982; Ginsburg, 1983; Schoenfeld, 1987).

Bednarz and Janvier (1988) presented the results of a 3-year longitudinal study using 39 children as they progressed from the first to the third grade. The main purpose of the study was to develop a constructivist approach to the concept of numeration and its learning, leading children to build a meaningful and efficient representation of numbers.

In order to evaluate the effects the constructivist approach had on children's understanding of numeration and difficulties encountered, Bednarz and Janvier were challenged to understand the way children were thinking. The researchers developed a constructivist approach that "helped children to progressively construct a significant and operational system for the representation of number" (p. 329). Of interest to the researcher was how students carry out operations involving groupings with concrete materials, and how students communicate representation of numbers. The constructivist "approach developed required a knowledge of children's thinking (difficulties, conceptions encountered by them), and continuous analysis of procedures and representations used by them in learning situations" (p. 302). Quantitative results revealed that the abilities and procedures developed by the children were still present. Most of the children using a constructivist approach were able to identify groupings and relations between them.

Researchers have recommended different teaching strategies to promote students' meaningful learning (Anderson, 1987; Anthony, 1996; Confrey, 1990; Minstrell, 1989; Novak & Gowin, 1984). Constructivism commits one to teaching students how to create more powerful constructions (Confrey, 1990). Confrey wanted to make a model of the practices of a teacher committed to constructivist beliefs. A Fundamental Mathematics Concepts class was chosen for the study. The study took place at a SummerMath program, an experimental summer program for young girls in high school. The focus of the study was on the teacher-student interactions. There were 11 students in the study ranging from 9th to 11th grades. In order to be in this program, students had to score less than 45% on a multiple choice placement test consisting of 25 items from the high school

curriculum. The students in the study worked in pairs on the curriculum materials provided each day. The purpose of the work was to suggest that alternative forms of instruction can exist in mathematics that differ in their basic assumptions from the tradition of "direct instruction" (p. 122). Data were taken from videotapes of the interactions. The method used was the idea of reflection in action. From the results, students were more persistent, more confident, and asked more questions in class. Follow-up evaluations of the program also indicated that students' scores on the mathematics part of the Standardized Achievement Test (SAT) improved as a result of the program.

An important tenet of constructivism is that learning is an idiosyncratic, active, and evolving process (Anthony, 1996). Anthony presented two case studies to understand the nature and consequences of passive and active learning behaviors in the classroom. He believed that "the nature of a student's metacognitive knowledge and the quality of learning strategies are critical factors in successful learning outcomes" (p. 349). Many believe that mathematics is most effectively learned through students' active participation. "Learning activities commonly identified in this manner include investigational work, problem solving, small group work, collaborative learning and experiential learning" (p. 350). Passive learning activities included "listening to the teacher's exposition, being asked a series of closed questions, and practice and application of information already presented" (p. 350). The purpose of the case studies was to provide a detailed description of how learning strategies are used in the learning context and to examine the appropriateness and effectiveness of each of the students' strategic learning behaviors. Data were collected throughout the school year by the

researcher using nonparticipant classroom observations, interviews, students' diaries, students' work, and questionnaires. For each case study, three lessons were recorded using two video cameras, creating a split-screen image of the teacher and student on a single tape. Each student was requested to view segments of the lesson and relive, as fully as possible, the learning situation. Although the case studies only illustrated contrasting learning approaches and acknowledged that much of the data reflected students' abilities to discuss their learning, the studies offered "an interesting perspective of learning processes used in the classroom and homework context" (p. 363). In each case study, the students believed they were pursuing learning in mathematics. The student in the first case study employed learning strategies appropriate for task completion rather than the cognitive objective for which the task was designed. The resulting mathematical knowledge and skills that the student acquired tended to be inert and available only when clearly marked by context. The student's learning was constrained by a limited application of metacognitive strategies. When faced with difficulties, the student provided little evidence that he was able to analyze, evaluate, or direct his thinking.

"Ineffective use of metacognitive control strategies such as planning, reviewing, reflection, selective attention and problem diagnosis meant that the student depended on the teacher and text to provide information and monitor his progress" (p. 364). The student in the second case study was able to adapt to the task the learning situation, and to maximize his learning opportunities. His learning environment was organized around goals of personal knowledge construction rather than goals of task performance. In both

case studies, the students contrasting learning approaches served "to illustrate the manner in which each student strives to cope in their world of experience" (p. 365).

Creating a learning environment where students have opportunities to negotiate; to challenge; and to question their own ideas, others' ideas, or the teacher's ideas can promote students' epistemological understanding about science (Tsai, 1998). This also may be the case for mathematics. Opportunities for children to construct mathematical knowledge arise as children interact with both the teacher and their peers (Davis et al., 1990). Many agree that meaningful learning for students could produce better cognitive outcomes, allow for more integrated knowledge, and foster greater learning motivation (Anthony, 1996; Cobb, Wood, & Yackel, 1991, 1992; Zazkis & Campbell, 1996). From a constructivist perspective, interactive learning with others is significant for effective learning (Bishop, 1985; Bruner, 1986; Clement, 1991; Cobb, Wood, & Yackel, 1990; Jaworski, 1992). Several researchers have emphasized that studies of children's learning should focus on the effects of children's interactions with other children (Cobb, Wood & Yackel, 1991; Tzur, 1999; Wiegel, 1998; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).

Science does not arise from a vacuum; rather, it comes from a complex interaction between social, technological, and scientific development (Tsai, 1998, p. 486). Tsai (1998) researched the importance of presenting the constructivist philosophy of science for students. He conducted a study to acquire a better understanding of the interaction between scientific epistemological beliefs and learning orientations in a group of Taiwanese eighth-grade students. Twenty eighth graders in a large urban junior high school were used in the study. Subjects were chosen using the following criteria: They

were above-average achievers, and they expressed a strong certainty and clear tendency regarding scientific epistemological beliefs (SEB) based on questionnaire responses. The questionnaire consisted of bipolar agree-disagree statements on a 5-1 Likert scale, ranging from empiricist to constructivist views about science. A qualitative analysis through interviewing the subjects revealed that students holding constructivist epistemological beliefs about science (knowledge constructivists) tended to learn through constructivist-oriented strategies when learning science, whereas students having epistemological beliefs, more aligned with empiricism (knowledge empiricists), tended to use more rote-like strategies to enhance their understanding. Knowledge constructivist subjects tended to have a more realistic view about the value of science, and their interest and curiosity about science mainly motivated them, whereas knowledge empiricist subjects were mainly motivated by performance on examinations. Although the study was not conducted using an experimental design, Tsai believed the results strongly suggested that "students' SEB played a significant role in students' learning orientations and how they organize scientific information" (pp. 485-486). The growth of science results from "interaction between science, technology, and society" (p. 486).

Zazkis and Campbell (1996) were concerned with the concept of divisibility and its relation to division, multiplication, prime and composite numbers, factorization, divisibility rules, and prime decomposition. They used a constructivist-oriented theoretical framework for analyzing and interpreting data acquired in clinical interviews with preservice teachers. The participants were 21 preservice elementary school teachers. The actions in the study included the following:

1. Examining preservice teachers' understanding of elementary concepts in number theory.

2. Analyzing and describing cognitive strategies used in solving unfamiliar problems involving and combining these concepts.

3. Adapting a constructivists-oriented theoretical framework.

Individual clinic interviews were conducted that probed participants' understanding of number theory concepts. The instrument was designed to reveal the participants' ability to address problems by recall or construction of connections within their existing content knowledge. "The results of the study suggested that in the schooling of the participants involved in this study, insufficient pedagogical emphasis has been placed on developing an understanding of the most basic and elementary concepts of arithmetic" (p. 562).

Cobb, Wood, and Yackel (1991) conducted a teaching experiment, where their primary focus was on understanding students cognitive development. The instructional activities included teacher-directed whole class activities and small group activities. In 1-hour sessions, the children worked in groups for 25 minutes, whole class instruction for 20 minutes, and 15 minutes for students to discuss with the class solutions and answers. During these instruction times, the teacher constantly moved about the class observing and frequently interacting with the students as they engaged in mathematical activities. The noise level was generally higher when the students were working with their partners. The teacher used a nonevaluative approach when students gave answers and solutions, even if their answer or solution was incorrect. Two video cameras were used to record the mathematics lessons for the school year. Analysis of "whole class dialogues and small group problem solving interactions focused on the quality of the children's mathematical activity and learning as they tackled specific instructional activities" (p. 161). The teacher's observations and analysis guided the development of instructional

activities. The teacher's purpose for whole class discussions was to encourage the children to verbalize their solution attempts. "The obligations and expectations mutually developed during whole class discussions also provided a framework for the children's activity as they worked in small groups, in that they were expected to solve problems in a cooperative manner and to respect each others' efforts" (p. 168). The children's level of conceptual understanding of mathematics influenced the social relationships that the children negotiated. Cobb, Wood, and Yackel concluded that the teaching experiment was reasonably successful. The children were able to solve mathematic problems in ways that were acceptable to them. "It was also apparent that the children's abilities to establish productive social relationships and to verbalize their own thinking improved dramatically as the year progressed" (p. 174). The researchers noticed the enthusiasm and persistence in the children. They also noticed that when the children were given challenging problems, they did not become frustrated but experienced joy working with those problems.

In 1992, Cobb, Wood, and Yackel explored the relationship between individual learning and group development of three 7-year-old students engaged in collaborative small group activity. They conducted a quantitative comparison of the control and treatment of students' arithmetic achievement, beliefs, and personal goals. There was no individual paper and pencil assignment and no grading of the students' work. Children began instructional activities by working collaboratively in groups followed by teacher-initiated discussions of mathematical problems, interpretations, and solutions. The materials and instruction strategies reflected "the view that mathematical learning is a constructive, interactive, problem solving process" (p. 99). The students learned as they

interactively designed situations for justification or validation, producing a controlled solution method.

A follow-up study was conducted with inner-city students with a predominately minority population. Cobb, Wood and Yackel found it necessary to conduct a quantitative comparison of arithmetical achievement, beliefs, and personal goals of the students. They found favorable results particularly with "students' conceptual development and problem solving capabilities in arithmetic, their perceptions of the reality of classroom life, and their personal goals as they engaged in mathematical activity" (p. 100). The children learned in classroom situations "as they interactively constituted situations for justification or validation" (p. 119).

Tzur (1999) conducted a constructivist teaching experiment with two fourth-grade students on the relationship of teaching and children' construction of a specific conception of their fraction knowledge. In a 3-year teaching experiment that began in the third grade and ended in the fifth grade, the author addressed the fourth-grade year in this study. Studying of fractions through teaching and learning contributed to the learning process in children's learning and the author's learning as the researcher-teacher. "The children's work during the study fostered an important transformation in their thinking about fractions" (p. 414). Teaching episodes were videotaped during the students' fourth-grade year. What generally took place in the teaching episode was children completing task to solve in Sticks. The researcher defined Sticks as a collection of possible actions that might be used to establish and modify fraction schemes. Students also were given opportunities to pose problem tasks to each other. The researcher realized that by allowing students to work this way enhanced their fraction schemes.

Wiegel (1998) conducted a study to investigate collaborative work with pairs of kindergarten students while they worked on tasks designed to promote early number development. The participants were 10 students in a public elementary school. The students in the study were selected based on the following: ability to work with someone, willingness to work with the researcher, and counting development skills. The 6-month study consisted of interviews with individual students and of teaching sessions with pairs of students. Students were interviewed three times during the study. The purpose of the interviews was to address student's understanding of standard number-word sequences and their ability to recognize and represent spatial patterns. The interviews were videotaped and audiotaped in the hallway outside the classroom. The teaching sessions involved activities focusing on the order of the number-word sequence, counting, and visual and auditory patterns. The teacher's role was to pose problems and facilitate the groups of students. As the sessions progressed, all students made some progress toward forms of organization requiring more coordination and toward increased involvement in the partner's actions. "Working in pairs supported and enhanced the students' cognitive development and promoted more sophisticated ways of social interaction" (p. 223). Wiegel also noted that the interactive working condition where students were paired homogeneously instead of heterogeneously provided learning opportunities different from whole-class, one-to-one teaching situations, or in small heterogeneous group settings. "For students who were able to reflect on and anticipate their actions, working in pairs led to cooperative ventures in which they solved counting tasks they were unable to solve alone" (p. 223).

Leikin and Zaslavsky (1997) discussed effects on different types of students' interactions while learning mathematics in a cooperative small-group setting. The study was conducted in four low-level ninth-grade classes consisting of 98 students. The main mathematics topic in the study was quadratic functions and equations, which is part of the Israeli secondary school mathematics curriculum. The topic was divided into six units. Each unit began with a whole class introductory lesson. The unit ended with a seventh lesson in which a unit test was given to all students. The four classes were divided into two groups (group 1a & 1b and group 2a & 2b) for comparison:

1. Students in group 1a learned all the material according to the experimental cooperative learning method.

2. Students in 1b served as a control group, learning all the material the conventional way.

3. In the second group, students in 2a and 2b learned by both methods, changing from one to another by the end of each unit.

The researchers collected data by observation, students' written self-reports, and an attitude questionnaire. Leikin and Zaslavsky noticed the following: (a) an increase in students' activeness, (b) a shift toward students' on task verbal interaction, (c) various opportunities for students to receive help, and (d) positive attitudes toward the cooperative experimental method. The results favored the experimental small-group cooperative method.

A number of studies also have documented that students can draw on their informal knowledge to give meaning to mathematical symbols when problems represented symbolically are closely matched to problems that draw on students informal knowledge (Fennema, Franke, Carpenter, & Carey, 1993; Lampert, 1986, 1990; Mack, 1990, 1995; Saenz-Ludlow & Walgamuth, 1998; Streefland, 1991). Saenz-Ludlow and

Walgamuth (1998) analyzed the interpretations of equality and the equal symbol of third-grade children who participated in a year-long, whole-class socio-constructivist teaching experiment. A socio-constructivist classroom teaching experiment was conducted primarily to understand students' conceptual constructions and interpretations. One of the major goals of this teaching experiment was to analyze the influence of social interaction on children's progressive understanding of arithmetical concepts. All of the students at the elementary school in which the teaching experiment took place were considered at-risk students. Twelve third-grade students participated in the study. Lessons were taped daily, and field notes of children's solutions were kept to analyze progressive activity and changes in the children's cognition. "The researcher's daily presence in the classroom allowed first-hand observation of the unfolding interaction and meaning-making processes of the students and teacher" (p. 158). The instruction these students received in first and second grades was characterized as traditional. The researchers believed that the teachers in the first and second grades emphasized performance and gave little consideration to the idiosyncratic ways in which children thought about numbers and mathematical symbols. "At the beginning of the third grade, even when the students could perform the addition algorithm correctly, they could give no justification for any of its steps indicating their lack of understanding of place value" (p. 162). The researchers' first objective was to remove students' procedural learning of operating with numbers by supporting their understanding of place value through activities with money exchanges and the packing of beans. When students were given the freedom to operate with algorithms, they were able to develop strategies different from conventional ones. At the completion of the study, Saenz-Ludlow and Walgamuth observed the following:

> 1. The children symbolized equality in different ways.
>
> 2. The dialogues indicate that these children initially interpreted the equal symbol as a command to act on numbers.
>
> 3. The dialogues also indicate the crucial role of classroom discussions on the children's progressive interpretations of the equal symbol.
>
> 4. The dialogues and the arithmetical tasks on equality indicate these children's intellectual commitment, logical coherence, and persistence to defend their thinking unless they were convinced otherwise.
>
> 5. The teacher's adaptation to the children's current knowledge and understandings was a determining factor in sustaining the children's dialogical interactions.
>
> 6. These children's dialogues raise our awareness of the cognitive effort entailed in the interpretation of and the construction of mathematical meanings from the conventional symbols, in particular the equal symbol. (pp. 184-186)

The children in the study expanded their conceptualizations of equality due to their active role in class discussions, the arithmetical tasks that took into account children's difficulties, and the teacher's ability to obtain a balance between teaching and the freedom to learn.

Mack (1995) conducted a study to examine the development of students' understanding of fractions during instruction. Four third-grade and three fourth-grade students received individualized instruction on addition and subtraction of fractions in a one-to-one setting for 3 weeks. The researcher, as teacher, gave instructions on a one-to-one setting. Each session lasted 30 minutes during regular school hours. Mack met with the students six times each for the 3-week period. The sessions, which included clinical interviews, were audiotaped and videotaped. The author was interested in helping students make connections mainly by posing problems and asking questions. Students

were given problems verbally and encouraged to think aloud while solving the problems. The results of the study led to the following conclusions:

> 1. Although students can draw on their informal knowledge to give meaning to mathematical symbols, they may not readily relate symbolic representations to informal knowledge on their own, even when problems presented in different contexts are closely matched.
>
> 2. Students' ability to relate symbols to their informal knowledge is influenced by their prior knowledge of other symbols.
>
> 3. As students moved from working with symbolic representations involving only proper fractions to working with representations involving both whole numbers and fractions, students explained the whole numbers represented symbolically in terms of inappropriate fractional quantities. (p. 438)

As students attempted to construct meaning for symbolic representations of fractions, they overgeneralized the meanings of symbolic representations for whole numbers to fractions, and they overgeneralized the meanings of symbolic representations for fractions to whole numbers.

Language and symbols play a role in constructivist accounts of mathematical development (Bednarz, Janvier, Poirier, & Bacon, 1993; Confrey, 1991; Kaput, 1991; Pirie & Kieren, 1994; Siegel, Borasi, & Fonzi, 1998). Siegel and others (1998) addressed the specific function that reading in conjunction with writing and talking can serve in mathematical inquiries to understand how inquiry experiences can be used in mathematics education. The participants in the study included a reading and mathematics educator and a group of secondary mathematics teachers. Data collection included teachers' written plans, photocopies of instruction materials and students' work, teachers' anecdotal records of what occurred during selected lessons, and audiotapes and

videotapes of classroom experiences. Based on the results, the researchers suggested that reading can serve multiple roles in inquiry-based mathematics classes and provide students with unique opportunities for learning mathematics. They also concluded that reading can make an important contribution to students' engagement in mathematical inquiries.

Constructivist perspectives on learning have been central to an empirical and theoretical work in mathematics education (Cobb, 1995; Simon, 1995; Steffe & Gale, 1995; von Glassersfeld, 1991) and, consequently, have contributed to shaping mathematics reform efforts (NCTM, 1989, 1991). Understanding learning through individual and social construction gives teachers a conceptual framework to understand the learning of their students (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991; Simon, 1991).

Cobb (1995) investigated the role that four second graders' use of particular instructional devices (e.g., hundreds boards) played in supporting their conceptual development based on reform recommendations of NCTM (1989). Cobb looked at students' use of the hundreds board for two reasons: (a) to observe the role instructional devices might have in supporting children's construction of numerical concepts and (b) to look at theoretical differences between constructivist and socialcultural conceptual development. "The investigation of children's use of the hundreds board provides an opportunity to explore the relationship between psychological constructivist and sociocultural accounts of mathematical development" (p. 363). The investigation took place in a second-grade classroom in the course of a year-long teaching experiment. Modes of children's early number learning were used to develop instructional activities.

The instructional strategy used was "small-group problem solving followed by a teacher-orchestrated whole-class discussion for children's interpretations and solutions" (p. 365). Videotapes of the mathematics lessons were done for an entire year. Manipulatives (e.g., hundreds boards) were made available to the children in all lessons involving arithmetical computation and problem solving. Students generally decided what available materials might help in solving a problem. The analysis Cobb presented focused on "individual children's cognitive construction of increasingly sophisticated place-value numeration concepts" (p. 379). The analysis indicated that the children's use of the hundreds board did not support the construction of increasingly sophisticated concepts of 10. "Children's use of the hundreds board did appear to support their ability to reflect on their mathematical activity once they had made this conceptual advance" (p. 377).

Simon (1995) presented data on a whole-class, constructivist teaching experiment in which problems of teaching practice required the teacher/researcher to explore the pedagogical implications of his theoretical (constructivist) perspectives. The 3-year study of the mathematical and pedagogical development of potential elementary teachers was part of a Construction of Elementary Mathematics (CEM) Project teaching experiment. Simon was interested in ways to increase these potential elementary teachers' mathematical knowledge and aid in their development of views of mathematics, learning, and teaching. There were 26 potential elementary school teachers involved in the study. Data were collected from a mathematics teaching and learning course, a 5-week prestudent-teaching practicum, and a 15-week student-teaching practicum taken by the participants in the study. The researcher took notes and videotaped the classes. There were no lecture class lessons, only small-group problem solving and teacher-led

whole-class discussions. The mathematical content of the mathematics teaching and learning course began with exploration of the multiplicative relationship involved in evaluating the area of rectangles. Simon believed that an advantage of the teaching experiment design was that time was built into the project to reflect on the understandings of the students. This teaching episode seemed to emphasize that the teacher did not create disequilibrium.

> The success of such efforts is in part determined by the adequacy of his model [meaning the teacher] of students' understanding. It also seems to support the notion that learning does not proceed linearly. Rather, there seem to be multiple sites in one's web of understandings on which learning can build. (p. 140)

Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz (1991) also conducted a study on students' response to constructivism in the classroom. They were interested in students' computational proficiency, conceptual development in arithmetic, personal goals in mathematics, and beliefs about reasons for success in mathematics. The instructional approach used in this teaching experiment was compatible with the constructivist view. The year-long study consisted of 338 second-grade students. The treatment group consisted of 187 students, and the remainder of the students were in the control group. The instructional activities designed to facilitate constructions of thinking strategies included activities from *What's My Rule* (Wirtz, 1977) and *War* (Kamii, 1985). At the end of the school year, two tests were administered to the study groups. One test was a state-mandated multiple choice standardized achievement test (ISTEP) and the second was a Project Arithmetic Test developed by the researchers. An ANOVA was conducted to determine the achievement of the treatment and control groups on the ISTEP and Project Arithmetic Test. "The comparison of students' performance on both

the ISTEP and Project Arithmetic Test indicated that the treatment students developed a higher level of reasoning in arithmetic than the control group" (p. 21). There was an indication that the teachers in the treatment group were successful partly because of their "development of classroom mathematics traditions in which their students could publicly express their thinking without risk of embarrassment" (p. 23). Students in the treatment group believed that success in mathematics stemmed from developing their own methods for solving problems. In contrast, students in the control group believed success was determined by using the same solution methods as the teacher or other students.

Nicholls, Cobb, Wood, Yackell, and Patashnick (1990) conducted a study on children's response to constructivism in the classroom. They were interested in second-grade children's reasons for success in mathematics. A series of questionnaires were administered to six second-grade classes to establish whether children's beliefs about the causes of success in mathematics were meaningfully related to their personal goals in mathematics. In conjunction with student's theories of success in school mathematics, the researchers were interested in measures that would determine how motivational influences of different mathematics teaching practices influenced students' success in mathematics. Only one class used teaching practices compatible with the NCTM (1989) recommendation that emphasis should be on establishing a climate that places critical thinking at the core of instruction. All statements should be subject to question, and both teacher and children should be open to reaction and elaboration from others in the classroom. The teaching practice consistent with a constructivist tradition in education was observed in class one. Class one was the only group consistent with either the NCTM recommendation or an approach to mathematics teaching compatible with

constructivism. In class one, after tasks were posed, students worked cooperatively in pairs to complete the tasks.

"One implication of constructivism is that mathematics, including arithmetical computation, should be taught through problem solving" (Cobb, Wood, & Yackel, 1991, p. 158). Franke and Carey (1997) conducted a study to investigate first-grade children's perceptions of mathematics in problem-solving environments. Two questions were addressed in the study:

> 1. What are children's perspectives on what it means to do mathematics in problem solving classrooms?
>
> 2. Do children in problem-solving classrooms with different demographic characteristics hold similar perspectives about what it means to do mathematics?

Individual interviews were conducted with 36 first-grade children. Six children, three girls and three boys, were randomly selected from each of six classrooms. The researchers were interested in determining children's perceptions about mathematics in problem-solving environments. Open-ended questions were asked to determine children's perceptions of what doing mathematics entailed. "The combination of open-ended, context-driven questions and children's ability to talk about their thinking allowed us more insights into young children's perceptions about mathematics" (p. 22). When asked about the use of manipulatives when doing mathematics, children responded that manipulatives were helpful in solving problems. The children recognized and accepted a variety of solution strategies, with many of the children valuing all solutions equally and assuming shared responsibility with the teacher and their peers for their mathematics learning. Children had varying perceptions of what it meant to succeed in mathematics.

The children in the study have helped the researchers to "see how critical it is to begin to understand children's perceptions of what it means to engage in mathematics" (p. 24).

Wood and Sellers (1996) conducted a class-by-instruction factorial design used to compare students in problem-centered classes for 2 years with students in problem-centered classrooms for 1 year, and with students in textbook classes for 2 years on a standardized achievement test. Also, the authors studied classes using problem-centered instruction for 2 years and compared them with students in problem-centered classes for 1 year on an instrument designed to assess students' conceptual development in arithmetic. Six classes received problem-centered mathematics instruction for 2 years in second and third grades. This instruction was generally reflective of a socioconstructivist theory of knowing and compatible with recommendations for reform in mathematics education. The researchers used an instrument designed to examine personal goals and beliefs about reasons for success in mathematics. Two tests were given to students to evaluate their arithmetic learning, the Arithmetic Test and the Personal Goals and Beliefs questionnaire. The results indicated that after 2 years of instruction in reform-based classes, students scored significantly higher on standardized measures of computational proficiency as well as conceptual understanding and they understood arithmetic concepts better than those in textbook-instructed classes. The students in the study believed "that it is important to find their own or different ways to solve problems, rather than conform to the method shown by the teacher" (p. 351).

One's representation of space is not a perceptual "reading off" of the spatial environment but is built up from prior active manipulation of the environment (Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998; Clements & Battista, 1992;

Clements, Swaminathan, Hannibal & Sarama, 1999). Battista and others (1998) examined students' structuring and enumeration of two-dimensional (2D) rectangular arrays of squares. The authors interviewed 12 second graders during the fall. Neither the research team nor the students' classroom teacher instructed students on enumerating rectangular arrays during the school year. The interviews were videotaped. After demonstrating how a plastic square inch tile fit exactly on a graphically indicated square given in a rectangle drawing, students were asked to do the following: (a) predict how many squares were in the rectangle, (b) draw where they thought the squares would be located on the rectangle, (c) then predict again how many squares would be needed, and (d) cover the rectangle with square tiles and determine again the number of squares needed. The results indicated that many students' understanding of the row-by-column arrays was unclear.

> In the traditional view of learning, it is assumed that row-by-column structure resides in two-dimensional rectangular arrays of squares and can be automatically apprehended by all. However, as we have seen in the present study, and consistent with a constructivist view of the operation of the mind, such structuring is not in the arrays--it must be personally constructed by each individual. (p. 531)

In a previous study focusing on three-dimensional arrays, Battista and Clements (1996) were only able to see consequences of a student's ability to perform coordinating actions. In their study, the researchers not only were able to see the consequences of a student's ability or inability to perform sequence of coordination actions used to organize a set of objects but they also were able to observe these actions, which gave them a better understanding of their characteristics.

Clements, Swaminathan, Hannibal, and Sarama (1999) investigated criteria preschool children used to distinguish members of a class of shapes from other figures. The participants in the study were 97 middle-class children ages 3 to 6 from two preschools and one elementary school with two kindergarten classes. The researchers investigated young children's methods used to distinguish geometric shapes common in the social-cultural environment. Data were collected using clinical interviews and paper-and-pencil tasks in a one-to-one setting. Children identified circles with a high degree of accuracy. Six-year-olds performed significantly better than did the younger children.

Battista (1999) used psychological and sociocultural components of a constructivist paradigm to provide a detailed analysis of how the students cognitive constructions are used to enumerate three-dimensional arrays of cubes developed and changed in an inquiry-based problem-centered mathematics classroom. There were six fifth-grade students in the study. The study took place over 4 weeks for 1 hour per day covering a unit on volume. The students had to find a way to predict correctly the number of cubes that would fill boxes described by pictures, patterns, or words. Each student in the study was given individual activity sheets to record answers. The teacher walked around the classroom, observing students interacting in pairs and encouraging collaborative communication. Students made predictions and checked their predictions by making paper boxes and filling them with cubes. "Having students first predict then check their predictions with cubes was an essential component in their establishing the viability of their mental models and enumeration schemes" (p. 442). This study showed how powerful learning can occur in problem-centered inquiry-based instruction. Cobb noted the following implications:

> Using the constructivist theory of abstraction permits an elucidation of students' need for repeated and varied opportunities to properly construct difficult concepts. Use of this theory also provides those insights into students' learning required for effective curriculum development and instruction because it describes precise constructive itineraries students follow in acquiring particular mathematical ideas. Second, instructional materials designed to promote intrapersonal (vs. interpersonal) cognitive conflict can be very effective in engendering accommodations and the resulting mathematics learning. (Battista, 1999, pp. 447-448)

Concrete Manipulatives for Teaching Mathematics

Many mathematics and science teachers in elementary and middle schools use manipulatives with varying degrees of success. Educators have argued that concrete manipulatives can help develop and enhance math problem solving and critical thinking skills, decrease anxiety, and increase concentration (Cuisenaire, 1992; Davidson, 1990; Langbort, 1988; Marzola, 1987; Ohanian, 1992; Scheer, 1985; Stone, 1987; Taylor & Brooks, 1986).

"One does not come to know a tool through a description of it, but only through its activity" (Confrey, 1990, p. 110). Reys (1971) gave the following suggestions on how concrete manipulatives can enhance the learning of students at all levels:

1. To vary instructional activities
2. To provide experiences in actual problem solving situations
3. To provide a basis for analyzing sensory data, necessary in concept formation
4. To provide an opportunity for students to discover relationships and formulate generalizations
5. To provide active participation by pupils

6. To provide for individual differences

7. To increase motivation related, not to a single mathematics topic, but to learning in general. (p. 555)

Burns (1996) suggested other common uses of concrete manipulatives:

1. Manipulatives help make abstract ideas concrete.

2. Manipulatives lift math off textbook pages.

3. Manipulatives build students' confidence by giving them a way to test and confirm their reasoning.

4. Manipulatives are useful tools for solving problems.

5. Manipulatives make learning math interesting and enjoyable. (p. 46)

Reys (1971) also suggested the following basic tenets for using manipulative materials in learning mathematics:

1. Concept formation is the essence of learning mathematics.

2. Learning is based on experience.

3. Sensory learning is the foundation of all experiences and thus the heart of learning.

4. Learning is a growth process and is developmental in nature.

5. Learning is characterized by distinct, developmental stages.

6. Learning is enhanced by motivation.

7. Learning proceeds from the concrete to the abstract.

8. Learning requires active participation by the learner.

9. Formulation of a mathematical abstraction is a long process. (p. 552)

In addition to Reys and Burns' list of ways to use manipulatives. Yeatts (1991) suggested other ways manipulative materials could help:

1. To introduce a new mathematical concept
2. To reinforce previous learning
3. To provide concrete representations of abstract ideas
4. To provide for individual learning styles
5. To foster creative thinking processes
6. To provide experiences in problem solving situations
7. To provide opportunities for students to become active participants in their own learning experiences
8. To provide opportunities for students to exchange viewpoints with their classmates
9. To diversify the educational activities
10. To enhance interest and motivation for learning new concepts. (p. 7)

In a meta-analysis of 60 studies from kindergartners to college-age students, Sowell (1989) found that long-term use of concrete manipulative material increased mathematics achievement and improved students' attitudes toward mathematics, when instruction using concrete material was presented by teachers knowledgeable about their use.

Many of the studies favoring the use of concrete material took place more than 20 years ago (Allen, 1978; Babb, 1976; Bledsoe, Purser, & Frantz, 1974; Bolduc, 1970; Bring, 1972; Brown, 1973; Callahan & Jacobson, 1967; Derderian, 1980; Earhart, 1964; Ekman, 1967; Fuson, 1975; Johnson, 1971; Letteri, 1980; Nichols, 1972; Nickel, 1971; Purser, 1974; Robinson, 1978; Romero, 1979; Stockdale, 1980; Tobin, 1974; Toney,

1968; Trueblood, 1968; Wallace, 1974; Wheatley, 1979). There have only been a few current studies found on the use of concrete manipulatives (Baxter, Shavelson, Herman, Brown, & Valadez, 1993; Burton, 1992; Canny, 1984; Chester, Davis, & Reglin, 1991; Gardner, Simons, & Simpson, 1992; McClung, 1998). It seems apparent that there is a gap in the literature on studies on the use of manipulatives and their effects on students' learning. This may be because manipulatives used to teach mathematics have been stressed many times and in different contexts, where it is assumed that using manipulatives in teaching mathematics is apparent (Behr, 1976).

Baxter and others (1993) were interested in the performance assessment and their effects on students in diverse groups. They developed a study with 105 Latino and Anglo sixth graders in two types of mathematics curriculum--hands-on and traditional. In the hands-on group of 40 students (13 Latino and 27 Anglo), the curriculum consisted of problem solving using manipulatives.

The instruction in the traditional group of 65 students (43 Latino and 22 Anglo) consisted of a textbook and worksheet curriculum. Performance assessment was comprised of several tasks, including measurement, place value, and probability. Due to time constraints, the hands-on group received only the measurement and place value assessment. The traditional group received all three assessments. The performance assessment consisted of describing objects, measuring length, and finding the area. The researchers concluded that the mean differences on the two performance assessments, where students with varying instructional histories could be compared, showed that students who had received hands-on mathematics instruction scored higher, on average, than students in the traditional curriculum. When comparing the groups by ethnicity, the

average scores for the Anglo students were higher than Latino students on all achievement measures. The extent of the difference varied, based on the instructional history of each student. The researchers concluded that assessments on mathematics performance relating closely to hands-on instructional activities provided reliable measures of mathematics achievement.

The manipulatives and strategies that young children would use if given free choice were of interest to Burton (1992). In the study, Burton explored young children's understanding of division. The subjects used in the study were 117 second-graders from five second-grade classrooms, three in one school and two in another school. The testing environment for the students in the study consisted of individual interviews conducted in the hallway outside the classroom. Each student was given an opportunity to explore the testing kit (manipulatives) before the interview began. The researcher was interested in how students would solve unfamiliar mathematical problems. Twelve problems presented one at a time were read and shown to the child during interview. The child decided which manipulative and strategy he or she wanted to use. The child's strategy and choice of manipulative were recorded for each problem. Each interview, on the average, lasted approximately 23 minutes. Some students took longer and some students took less time to answer the questions. All of the manipulatives in the kit were used at least once by one child, in spite of the 38 children that did not use any manipulatives on any of the 12 questions. Burton concluded that "second-grade children are capable of solving some division word problems if manipulatives are supplied. They are especially successful in this if the objects match the problem of context" (p. 13).

Many classroom teachers focus on covering the material in the textbook from an abstract approach despite the support in the literature for the use of manipulative materials. Canny (1984) was interested in determining if manipulative materials had a greater impact when used to introduce or reinforce a concept or both. The researcher investigated the role of manipulative materials in improving achievement in computation, concept-formation, and problem solving in fourth-grade mathematics. There were 123 fourth graders included in the study. The students were divided into four groups. Group A teachers used manipulative material to introduce the concepts. Group B teachers used manipulatives for reinforcement after the traditional teaching method. Group C teachers used manipulatives to introduce the concept, practiced, and reinforced the concept using the textbook and manipulatives. Group D teachers only used the textbook; they did not use manipulative materials for the mathematics lessons. The classroom teachers implemented the lessons to avoid as much disruption of the classes as possible. The students in the study were given The Science Research Associates (SRA) achievement test, a researcher-designed written achievement test and retention test that matched the lessons and the textbook. The researcher conducted in-service sessions and demonstrated teaching lessons because the teachers in the study had little experience with using concrete objects to aid in learning. An ANCOVA model with multiple contrasts was used in the study.

Canny concluded that "the group using manipulative materials for only reinforcement scored the lowest while the group using manipulatives for only introduction scored the highest" (p. 71). Most of the test results in this study provided additional support for the research favoring the use of manipulative materials in teaching

and learning mathematics. The overall findings of this study would influence the researcher to use manipulative materials to introduce concepts because it produced the highest scores on the SRA test and the retention test.

"Using math manipulatives is one method of teaching that makes learning real to students" (Chester and others, 1991, p. 4). Manipulatives help students to become actively involved in doing mathematics instead of just listening to lectures and completing paper and pencil activities. Chester and others investigated the effect of manipulatives on increasing mathematics achievement of third-grade students. The students in the study consisted of two randomly selected third-grade classes with 26 students in each class. Before the start of the study on a geometry unit, the students in the study were given a multiple choice unit test. The researchers used an ANCOVA and t-test to analyze the data. Manipulatives were used to teach concepts in the geometry unit during the 2-week hourly teaching sessions. Drawings and diagrams were the only things used to teach the concepts in the control group. There was a significant difference between the pretest and posttest scores of both groups. Chester and others concluded that "the experimental group, which used the math manipulatives, received higher adjusted posttest scores than the control group, which used only the textbook" (p. 17).

McClung (1998) conducted a study to determine if the use of manipulatives in an Algebra I class would make a difference in the achievement of the students. The researcher used two Algebra I classes mixed with 10th and 11th graders. One group had 24 students and the other group had 25 students. Group A was the control group, and the students in this group were taught using traditional teaching methods consisting of lecture, homework, and in-class worksheets. Experimental Group B was also taught

using the traditional teaching method but used manipulative Algeblocks instead of the worksheets. A pretest was given prior to the beginning of the study to assure homogeneity. A posttest, identical to the pretest, was given at the end of the study. A two-sample t-test was used to analyze the data. There was a significant difference between the two groups. The control group's mean score, using no manipulatives, was higher than the experimental group's mean score, using manipulatives. This implied that students using the traditional teaching methods outperformed the students using the manipulatives. According to Piaget (1970), the concrete operational stage (7 to 12), is the foundation for the use of manipulatives. One reason students using manipulatives may have scored lower than students not using manipulatives could be because students used manipulatives in the class lessons but were not permitted to use them on the posttest. Although no information was given about the instructor's knowledge on manipulatives, a second reason may be that the instructor had limited knowledge of the concept of using manipulatives and did not obtain this information before the study began.

Hinzman (1997) conducted a study to determine the differences in mathematic scores/grades of middle school students after the use of hands-on manipulatives and group activities. Thirty-four eighth-grade students in two prealgebra classes participated in the study. The instruments used in the 18-week study included a pretest, observation of the students participating in the lessons, and a student attitude survey. Chapter tests were given to the study groups to determine the effects of manipulative use on test performance. Notebook tests were also given to determine the effects of homework on class performance. A T-test was used to compare the test results of the two prealgebra classes. There was no control group used in the study.

Hinzman concluded that students' performance during in class instruction improved with the use of manipulatives. The study did not show any significant changes in grades for students who used manipulatives and students who did not use manipulatives. However, based on the results of the student attitude survey, there was a significant improvement of students' attitude towards mathematics.

There was significant interest in determining whether combining computer-aided instruction with hands-on science activities would increase students' cognitive and affective assessment outcome (Gardner and others, 1992). Gardner and others used a "weatherschool" meteorology program to determine the effect of computer-aided instruction in the elementary school classroom. The instruments used to measure students' cognitive and affective domains were paper and pencil tests. The first test contained 13 questions and measured the knowledge and application level. The second test contained 15 questions and measured students' attitudes toward science and computers. There were three treatment groups. The first treatment group with 47 students included hands-on activities. The second treatment group with 46 students included a combination of hands-on activities and the weatherschool program. The third treatment group with 21 students used text based learning only. All three treatment groups met for 10 days for two class periods each day. The results of the study indicated that students' attitude and knowledge toward science increased with hands-on activities. The pretest and posttest scores from Treatment Group II showed higher gains than Treatment Group I. Treatment Group III scored higher on the pretest for conceptual assessments. However, they scored lower than both groups on the posttest. "Hands-on

activities, CAI, or both apparently lead to increased understanding and more positive attitudes" (p. 336).

Not all researchers would agree that concrete manipulatives have significant effects on students' learning (Labinowicz, 1985; Resnick & Omanson, 1987; Thompson, 1992). Thompson was interested in the effects of concrete manipulatives on student's understanding of whole number concepts. He investigated students' engagement in tasks involving the use of base-ten blocks and their contributions to students' construction of meaning of decimals. The study consisted of 20 fourth-grade students, 10 boys and 10 girls. Students were assigned to one of two treatment groups based on result of their pretest scores. Students with similar scores were put in opposite groups. One treatment group, taught by their regular classroom teacher, used Blocks Microworld in instruction. The other treatment group, taught by a research assistant, used wooden base-ten blocks. The microworld instruction was videotaped. The wooden base-ten blocks instruction could not be videotaped because the class had two special education students in attendance who were not part of the study. The teacher in the microworld experiment group used a computer connected to a large screen projector for whole-class discussion. The teacher in the wooden base-ten blocks experiment group used the overhead projector and plastic blocks for whole-class instruction. Both teachers in the treatment groups used a highly detailed script for instruction, which restricted their actions. This type of instruction was used to get more responses from the students. At the end of the 9-day study, students were administered a posttest containing items from the pretest and additional items on decimal concepts. Thompson concluded that there were no significant changes in either group regarding whole number concepts.

Tanbanjong (1983) conducted a study on the effects of using manipulative material in teaching addition and subtraction to first-grade students. The 6-week study consisted of 350 first-grade students. Half of the students were assigned to the treatment group and the other half were assigned to the control group. The instructional activities covering content in addition and subtraction of one-digit and two-digit numbers included activities from a textbook provided by the Ministry of Education of Bangkok, Thailand, for first-grade students. Students in both groups received mathematics instruction for 40 minutes per day for 6 weeks. In the treatment group, the instructor used the following manipulatives: a pocket chart, a set of trading chips, a bundle of sticks, and an abacus. In the control group, there were no manipulative materials used, only traditional teaching methods. The researcher observed each group every week. A 20-item posttest, used to measure students' achievement, was given at the conclusion of the study. A one-way ANOVA was used to analyze the data. The children in the treatment group scored significantly higher on the achievement test than the children in the control group. The researcher concluded that the manipulative approach was more effective than the traditional teaching method in facilitating achievement gains.

Kanemoto (1998) also conducted a study on the effects of manipulatives on students' learning. The researcher was interested in the effects of using manipulatives on fourth-grade students in learning place value. The 4-week study consisted of 51 fourth-grade students from two different classes. The treatment group contained 26 students, and the control group contained 25 students. A pretest was given to both groups to make sure there were no significant differences in the groups. The treatment group was taught place value using a combination of manipulatives and textbook

instruction. The control group was taught place value using textbook instruction only but no manipulative material. There were 11 one-hour mathematics lessons conducted during the study. A 20-item posttest was given at the completion of the study. A chi-square test was used to analyze the data. The researcher concluded that the results of the posttest showed that the use of manipulatives did not provide a difference in mathematics achievement between the treatment and control groups when studying place-value.

Harris (1993) also conducted a study to determine the effects of the use of manipulative materials on the development of mathematical concepts and skills. The 12-day study consisted of 331 seventh-grade students. The classrooms were already intact; therefore, random assignment was not possible. The control group consisted of 160 students, and the remainder of students were in the treatment group. Both groups were given The California Achievement Test as a pretest to measure broad concepts of a specific content area. All classes in the study used the same mathematics textbook. Eleven lessons designed by the researcher, were developed for both groups. The lessons covered objectives relating to perimeter and area. The control group received traditional lecture, demonstration, and paper and pencil instruction. The treatment group received the same instructional lessons except their instruction included the use of the following manipulatives: geoboards, rubber bands, rulers, tape measures, trundle wheel, overhead tiles, unifix cubes, square tiles, dot paper, and grid paper. At the end of the study, two tests were administered to the two groups. One test was the California Achievement Test (CAT), and the second test, consisting of multiple-choice and open-ended questions, was a teacher-made unit test developed by the researcher. An ANOVA was

conducted to determine the achievement of the treatment and control group on the CAT and the teacher-made unit test. The comparison of students' performance on both the CAT and the teacher-made unit test indicated that the students in the treatment group developed a better understanding of perimeter and area than the control group. Students in the treatment group posited that manipulatives helped them to understand perimeter and area and to apply the formulas correctly. No response was given for the control group.

Technology and Simulated Manipulatives in the Classroom

An instructional model for teaching mathematics to young children should emphasize active learning and problem solving with concrete materials (NCTM, 1989). "Technology must support and enhance this model" (Campbell & Stewart, 1993, p. 252). Technology education at the elementary level begins with three things in mind: the child, the elementary school curriculum, and the appropriate technology activity (Kirkwood, 1992). Implementing technology in the mathematics education classroom helps "promote high level thinking skills and support concept development" (Harvey & Charnitski, 1998, p. 157). Campbell and Stewart (1993) stated that "perspective for using technology with young children is that these tools permit teachers and children to investigate the richness of mathematics" (p. 252).

An important role for computers is to assist students with exploring and discovering concepts, with the transition from concrete experiences to abstract mathematical ideas (NCTM, 1989). "In computer environments, students cannot overlook the consequences of their actions, which is possible to do with physical manipulatives"

(Clements & McMillan, 1996, p. 274). Clements and McMillan gave the following advantages to using the computer instead of physical manipulatives:

1. They avoid distractions often present when students use physical manipulatives.

2. Some computer manipulatives offer more flexibility then do their non-computer counterparts.

3. Another aspect of the flexibility afforded by many computer manipulatives is the ability to change an arrangement of the data. Primary Graphing and Probability Workshop (Clements, Crown, & Kantowski, 1991) allows the user to convert a picture graph to a bar graph with a single keystroke.

4. Students and teachers can save and later retrieve any arrangements of computer manipulatives. Students who had partially solved a problem can pick up immediately where they left off.

5. Once a series of actions is finished, it is often difficult to reflect on it. But, computers have the power to record and replay sequences of actions on manipulatives. This ability encourages real mathematical exploration.

6. The computer connects manipulatives that students make, move, and change with numbers and words.

7. Computer manipulatives dynamically link multiple representations. For example, many programs allow students to see immediately the changes in a graph as they change data in a table.

8. Students can do things that they cannot do with physical manipulatives. For example, students can expand computer geoboards to any size or shape.

Computers also serve as a catalyst for social interaction (Clements, 1998). Social interaction constitutes a crucial source of opportunities to learn mathematics (Piaget, 1970).

Connell (1998) conducted a study to determine the impact for technology use in a constructivist elementary mathematics classroom. Students were randomly assigned to one of two conditions. Twenty-five of the students were assigned to the technology-aligned classroom (TAC), where the teacher used the computer as a student tool for mathematics exploration. Twenty-seven students were assigned to the technology-misaligned classroom (TMC), where the teacher used the computer as a presentation tool. Daily teacher instruction was the treatment in the study. "Technology was present in both classrooms, but only used as an integral portion of the mathematics instruction in the TAC" (p. 325). Both classrooms used a similar constructivist mathematical pedagogy with similar set of materials. Technology "was used in a constructivist fashion" (p. 331), not as an afterthought. Both groups were successful in learning the required grade level mathematics. "It is clear that the constructivist approaches to instruction as utilized in this study were effective" (p. 331).

There is an assumption that computer software produces the same cognitive effect as touching physical mathematics manipulatives (Terry, 1995). Terry conducted a study to determine the effects that mathematics manipulatives (MM) and mathematics manipulative software (MS) had on students' computation skills and spatial sense. The 4-week study consisted of 214 students in Grades 3, 4, and 5. Sixty-eight students were in the MM group; 68 students were in the MS group; and the remainder of students were in the mathematics manipulative with mathematics manipulative software (BOTH) group.

Base ten blocks and attribute shapes, both in concrete manipulative and computer manipulative form, were used for the computation and spatial sense lessons. At the end of the study, two tests, designed by the researcher, were administered to the three groups. One test covered computation, and the second test covered spatial sense. A three-way ANOVA was conducted to determine the achievement of the three groups on the computation and spatial sense test. The comparison of students in the three groups on the spatial sense test indicated that there was no significant difference. The comparison of the three groups on the computation test indicated that the students using both concrete manipulatives and computer manipulatives developed a better understanding of computation than the other two groups. The researcher felt that the use of both was the most effective way to teach computation.

Kim (1993) also was interested in the effects of concrete manipulatives and computer-simulated manipulatives on students' learning. Kim conducted a study on young children to determine the effects computer-simulated and concrete manipulatives would have on their learning of geometric and arithmetic concepts. The 6-week study consisted of 35 kindergarten children. Seventeen students were in the hands-on group, and the remainder of students was in the computer simulation group. The instructional activities for both groups included activities with geoboards, attribute blocks, and Cuisenaire rods. There was no control group in the study. The researcher was concerned with possible differences in initial intellectual ability and maturity, because the two treatment groups were not randomly assigned. There were 12 lessons, with each lesson lasting 25 minutes. At the beginning of the three units a pretest was administered, and at the end of each unit, a posttest was given to determine students'

progress in the three units. The first unit covered geometry; the second unit covered classification; and the third unit covered seriation, counting, and addition. An ANCOVA was conducted to determine the achievement of the treatment groups on the unit tests. The results indicated that hands-on and computer-simulated manipulatives were similar in their effectiveness. Both groups made significant gains from the pretest to the posttest on all three unit tests. The researcher concluded that using either concrete or computer-simulated manipulatives helped children learn mathematical concepts relating to geometry, seriation, counting, and addition. The researcher also believed that hands-on and computer-simulated manipulatives were equally effective for teaching mathematics concepts to young children.

Berlin and White (1986) studied the effects of combining interactive microcomputer simulations and concrete activities on the development of abstract thinking in elementary school mathematics. The sample of students was randomly selected from two elementary schools. Three levels of treatment were administered to the 113 students in the study: (a) concrete only activities, (b) combination of concrete and computer simulation activities, and (c) computer-simulation only activities. Two paper and pencil exams requiring reflective abstract thought were administered to all the subjects. The researchers felt it was still unclear what effects concrete manipulation of objects and computer simulations had on students' understanding. The researchers concluded that effects of concrete and computer activities produced different results, depending on the socio-cultural background and gender of the child. Berlin and White posited more research was needed to determine the effect of concrete and computer activities in the learning of elementary school mathematics.

Researchers have also found computer simulations useful in the science curriculum (Bork, 1979; Lunetta & Hofstein, 1981). Choi and Gennaro (1987) conducted a study that compared the effects of microcomputer-simulated experiences with a parallel instruction using hands-on laboratory experiences for teaching volume displacement concepts. The subjects consisted of 128 eighth-grade students in earth science classes. The method used in the study was an experimental design with a control group. The students were randomly assigned to one of two treatment groups--the microcomputer-simulated group (the experimental) and the hands on laboratory group (the control). The experimental group used a computer program which included graphics, animation, and color for visualizing concepts; blinking words to help students focus on important instructions; and immediate feedback to enhance learning. The control and the experimental groups conducted parallel experiments, but the experimental group used worksheets and hands-on experiments. The experiments took 55 minutes in the control group and only 25 minutes in the experimental group. After the 2-day experiment, a 20-item multiple choice posttest was administered to students in both groups to measure the students' understanding of concepts related to volume displacement. There was no pretest given. The results of the study showed that computer-simulated experiments were as effective as hands-on laboratory experience. Forty-five days later a retention test was given to the students in both groups. A three-way ANOVA was used to interpret the data. Choi and Gennaro found that the computer-simulated experiences were as effective as the hands-on laboratory experiences. Also, there was not a significant difference in retention scores of the two groups.

One implication of this study was that "computer simulations could be used for the teaching of some concepts without the extra needed effort and time of the teacher to prepare make-up material" (p. 550). Another implication was that the computer-simulated experience "enabled the students to achieve at an equal performance level in approximately one-fourth the time required for the hands-on laboratory experiences (25 minutes compared to 95 minutes)" (p. 550).

Probability

Because of the emphasis on probability learning in the school curriculum, there is a need for further on-going research on the learning and teaching of probability (Shaughnessy, 1992). The study of statistics and probability stress the importance of questioning, conjecturing, and searching for relationships when formulating and solving real-world problems (NCTM, 2000). We live in a society where probabilistic skills are necessary in order to function. Probability describes the world in which we live. Many everyday functions depend on knowing and understanding probability. Milton (1975) suggested the following reasons for introducing probability as early as the primary level:

> 1. The basic role which probability theory plays in modern society both in the daily lives of the public at large, and the professional activities of groups within the society, e.g. in the sciences (natural and social), medicine and technology.
>
> 2. Probability theory calls upon many mathematical ideas and skills developed in other areas of school coursed, e.g. set, mapping, number, counting, and graphs.
>
> 3. Students are able to work in a branch of mathematics, which is relevant to current activities in life. (p. 169)

Jones, Langrall, Thornton, and Mogill (1999) conducted a study to evaluate the thinking of third-grade students in relation to an instructional program in probability. The focus of the evaluation was on student learning. The design of the instructional program was consistent with a constructivist orientation to learning. "Opportunities to construct probability knowledge arise from students attempt to solve problems, to build on and recognize their informal knowledge, and to resolve conflicting points of view" (p. 492). There were two third-grade classes involved in the study: one was an early-instruction group, in the fall, and the other one was a delayed-instruction group, in the spring. The instructional program consisted of 16 40-minute sessions with two sessions occurring each week for 8 weeks. The probability problem task was based on the following essential constructs: "sample space, probability of an event, probability comparisons, and conditional probability" (p. 494). Problems were chosen so they would be accessible to students functioning at different levels. The most important feature of the data was the number of students in both groups that increased their informal quantitative level of probabilistic thinking following instruction. An important finding was that 51% of the students who began instruction below an informal quantitative level reached this level after instruction. A repeated measures ANOVA was used to evaluate the overall effect. This analysis "demonstrated that both the early and delayed instruction groups showed significant growth in performance following instruction" (p. 517).

Analyses of the students' learning during the instructional program, which revealed a number of patterns that appear to have critical ramifications for children understanding of probability, are related to the following:

1. children's conception of sample space

2. their use of part-part and part-whole relationships in comparing and representing probabilities

3. their use of invented or conventional language to describe their probability thinking. (p. 515)

Many of the students revealed higher level sample-space thinking following instruction. The results of this study showed that the probabilistic thinking framework could be used to develop an effective instructional program in probability. A further conclusion of this study was that the use of part-part reasoning provided a threshold level for dealing with probability situations that incorporated some need for quantitative reasoning. The researchers suggested that further research is needed to help classroom teachers use the probabilistic thinking framework to enhance student learning.

Fishbein and Gazit (1984) analyzed the effect teaching of probability indirectly has on intuitive probabilistic judgments. The researchers defined intuition as "a global, synthetic, non-explicitly justified evaluation or prediction" (p. 2). The study evolved around the indirect effect of a course on probability on typical, intuitively-based probabilistic misconceptions. The teaching program designed for students in Grades 5, 6, and 7 was intended to determine the capacity of their ability to assimilate, correctly and productively, basic concepts and solving procedures in probability. The students were given various situations that gave them opportunities to be active in the following areas:

1. calculating probabilities

2. predicting outcomes in uncertain situations

3. using operations with dice, coins and marbles for watching, recording and summing up different sets of outcomes. (p. 3)

Two questionnaire assessments were used to determine the teaching effects. The first questionnaire was used to determine to what extent pupils in the experimental class had assimilated and were able to use concepts and procedures taught. The purpose of the second questionnaire was to determine the indirect effect of instruction on children's intuitively based misconceptions. The first questionnaire was too difficult for fifth graders. Nevertheless, for sixth and seventh graders, progress was evident. Because of the low negative results for fifth-grade with questionnaire one, the second questionnaire was not reliable and relevant enough for this grade level. However, "in grades six and seven, the teaching programme has had an indirect positive effect on their intuitive biases" (p. 22).

Vahey (1998) was interested in middle school students' understanding of probability. In his dissertation, he used a Probability Inquiry Environment (PIE) curriculum. The PIE curriculum consisted of computer-mediated inquiry-based activities, hands-on activities and whole-class discussion. Vahey used a quasi-experimental design with four seventh-grade classes. Two of the classes used the PIE curriculum for 3 weeks and the other two classes used a teacher-designed curriculum for 3 weeks. The same teacher taught the 3 week curriculum for both groups. The students in both groups were given the same quantitative pretest and posttest. There was no significant difference between the groups on the pretest. However, a significant difference was found between the two groups on the posttest. The students using the PIE curriculum out-performed the students in the comparison group on the posttest. Vahey posited that students using the PIE curriculum could successfully build on their existing

understandings by engaging in activities before formal introductions of concepts, definitions, and terms.

Drier (2000) was also interested in children's probabilistic understanding. In her dissertation, she conducted a 2-month exploratory teaching experiment with three 9-year-old children at the end of their third-grade year. Each child in the study participated in 10 hours of teaching sessions using a computer microworld program designed by the researcher. The computer microworld program, *Probability Explorer*, was designed for students to explore with probability experiments. A pretest and posttest interview assessment was given to all three children. All teaching sessions were videotaped and audiotaped. The children's development of probabilistic reasoning and their interaction with the computer tools varied during the study. The results of the study indicated that open-ended computer tools could represent children's development of probability concepts. Drier posited that further research was needed with *Probability Explorer* in a variety of small groups and classroom situations with different task and technological versus nontechnological learning on students' reasoning and understanding of probability. She also recommended that further research be done to help students connect their "understanding of probability with their understandings of other mathematics concepts and their everyday experiences" (p. 359).

Jiang (1993) also developed a simulated computer program, *CHANCE*. The program was used to eliminate the use of physical materials when teaching and learning probability. Without being able to generate large number experiments, it is difficult for students to believe that certain events have the same possible chance of occurring (Jiang & Potter, 1994). Jiang and Potter conducted a teaching experiment to evaluate the

performance of *CHANCE*. The researchers were interested in knowing if the use of *CHANCE* could effectively help students achieve conceptual understanding of probability. There was no control group due to time constraints. Like the aforementioned study, the teaching experiment was exploratory, and a small sample was used to obtain information that was more detailed. Three boys and one girl participated in the study. They were in 5th, 6th, 8th, and 11th grades, respectively. The students in Grades 5 and 6 were in group one and the students in Grades 8 and 11 were in group two. The teaching experiment consisted of two 1-hour sessions per week for 5 weeks. The children in the study took the same pretest and posttest. The pretest was administered to determine students' previous knowledge. The posttest was used to determine both students' knowledge and students' ability to solve problems using simulations and modeling. They found *CHANCE* useful in making probability instruction meaningful, stimulating and increasing students' learning difficulties, and in helping students overcome their misconceptions about probability. Generalizability was limited due to the small sample used in the study. Also, because the students were above average, there was no way to determine if students below average would benefit or if the positive effect of *CHANCE* was due partly because of the beginning academic level of the students.

Fischbein, Nello, and Marino (1991) conducted a study on students' difficulties with probabilistic concepts. For example, the concept of certain events is more difficult to comprehend than that of possible events. They used 618 students in three groups. In the first group, there were 211 students in Grades 4 and 5. In the second group, there were 278 students in Grades 1, 2, and 3 with no prior instruction in probability, and in the third group, there were 130 students in Grades 1, 2, and 3 with prior instruction in

probability. Students answered one of two parallel 14-item questionnaires (A or B) of probability problems. The items on the questionnaires required the subjects to explain their answers. On many of the problems, the subjects had to determine how the sample space related to a certain event. The authors indicated that students had difficulty answering the probability problems on the questionnaire because of students' lack of skills for rational structures.

Fast (1999) also noticed that students overcame misconceptions in probability after instruction. Fast investigated the effectiveness of using analogies to effect conceptual change in students' alternative probability concepts. He wanted to determine if using analogous anchoring was an effective approach to overcoming probability misconceptions. Anchors are analogies or examples used to simplify difficult concepts. It was difficult generating anchoring situations that would be effective for the students. The investigation centered on high school students' probability concepts. Most of the students in the study felt analogies were useful in reconstructing their thinking. Generally, misconceptions are "highly resistant to change" (p. 234). Thus, "the results of the study showed that the use of analogies was effective for reconstructing high school students' probability misconceptions" (p. 234). Fast concluded that the use of analogies could make a valuable contribution to overcoming probability misconceptions.

Summary

There is much support on children learning concepts of probability as early as elementary school. Many important documents, such as *Principle and Standards* (2000), have been revised to include the teaching and learning of probability as early as kindergarten. Although few studies exist at the elementary level, researchers recognize

the importance of introducing probability at an earlier age (Shaughnessy, 1992; Vahey, 1998).

In the review, many of the studies (e.g., Fishbein & Gazit, 1984; Baxter and others, 1993; Jiang, 1993; Vahey, 1998) approached the concepts of probability and statistics starting in the middle grades and higher through computer simulations and concrete manipulatives. It was evident in a few of the studies (e.g., Choi & Gennaro, 1987; Kim, 1993) that the use of computer simulations was helpful for learning mathematics and science and providing more learning time for students. Some of the studies (Jones and others, 1999; Drier, 2000) provided evidence that students may learn probability concepts earlier than the age predicted by Piaget and others. At this particular time, there are very few studies reported on the investigation of how computer simulations and concrete manipulatives can benefit elementary students in learning probability concepts. It is not clear at this point the effects that concrete manipulatives and computer simulations have on elementary students' understanding probability concepts at an early age.

In many of the studies (Mack, 1995; Anthony, 1996; Frank & Carey, 1997; Wiegel, 1998; Drier, 2000, interviews and case studies were used to obtain information. Some researchers (e.g., Choi & Gennaro, 1987; Jiang, 1993; Kim, 1993) did not establish control groups or validate their instruments. Many of the instruments were used to provide information on students' probabilistic thinking and misconceptions about probability. In some quantitative studies (e.g., Confrey, 1990), researchers used nonrandom samples which were often a consequence of school-based research. In addition, generalizability was not possible due to the small number of students used in

these studies, which may have weakened statistical results. It is still important to consider the results of these studies because they can serve as a starting point for further investigation. In some studies, researchers designed their own computer program (e.g., Jiang, 1993; Drier, 2000) for teaching probability with computer simulations because they posited there was inadequate software available. One program found to be the most promising to use for teaching probability using computer simulations was *Probability Explorer*. However, there has been no research conducted on its effect on whole-class learning.

Some studies revealed that when examining children about specific concepts in probability, many misconceptions regarding children's understanding of probability were evident. For example, some students may have misconceptions about a coin toss being fair if they believed that getting a tail was best.

The constructivist theory has been prominent in recent research on mathematics and science learning. The constructivist learning and teaching theory has influenced the instructional environment. Because of this theory, many studies (e.g., Bednarz & Janvier, 1988; Confrey, 1990; Anthony, 1996; Schulte, 1996) have stressed the importance of students taking an active role in their learning process. The constructivist learning theory emphasizes the importance of the knowledge, beliefs, and skills an individual brings to the experience of learning. It also provides a basis for recent mathematics education reform efforts. There is a need for further research for teaching and learning probability from a constructivist perspective, especially at the elementary level.

CHAPTER 3
METHODOLOGY

Overview of the Study

This chapter describes the research objectives, the development of the research instruments, and the participants for the study. It outlines the procedures for the research design and the data analysis.

The purpose of this study was to investigate the impact of using computer-simulated manipulatives and concrete manipulatives on elementary students' probability learning skills and concepts of experimental probability. Of interest to the researcher was students' ability to predict outcomes of simple experiments and to discuss and describe the likelihood of events using words such as certain, likely, unlikely, and impossible. The researcher was also interested in incidental fraction learning that might occur as participants engage in probability experiences.

Research Objective

The following null hypotheses were tested in the study:

1. There will be no significant difference between students who use computer-simulated manipulatives and students who do not use computer simulated manipulatives regarding students' learning skills and concepts in experimental probability.

2. There will be no significant difference between students who use concrete manipulatives and students who do not use concrete manipulatives regarding students' learning skills and concepts in experimental probability

3. There will be no interaction between students who use concrete manipulatives and computer-simulated manipulatives regarding students' learning skills and concepts in experimental probability.

Description of the Research Instruments

Experimental Probability Instrument

The researcher designed the Experimental Probability Instrument (EPI) (see Appendix A) and administered it as a pretest and posttest. The EPI was used to measure students' probability learning skills and concepts of experimental probability. The research was conducted to examine the effects of employing computer-simulated manipulatives and concrete manipulatives on the above factors.

To insure that the content of the instrument reflected content domain of elementary students regarding the concept of probability, the researcher used several methods to determine the content validity of the instrument. First, the researcher used the literature as a guide to locate items specific to probability. Secondly, the researcher's teaching experience was used to examine the items for appropriate content. In addition, items used to create the EPI were adapted from Konold, Pollatsek, Lohmeier, and Lipson (1993), Vahey (1998), and Drier (2000). The objectives of the EPI are given in Table 3. Fourthly, the researcher submitted the instrument to two mathematics educators for their review and analysis of content and later received written comments from them regarding the content and objectives of the instrument. They provided feedback on the wording and numbering of some of the items and drawings of the objects the researcher incorporated into the design of the instrument.

Table 3

Objectives of Experimental Probability Instrument

# of items	Objective: Test the students understanding, learning skills, concepts and ability to . . .
9	decide whether a game is fair or unfair by specifying the estimated probability of an event, given the data from an experiment
6	discuss the degree of likelihood by distinguishing whether an event is an instance of certainty, uncertainty, or impossible
3	test predictions and identify estimates of true probability given a set of data from an experiment, by counting the number of outcomes of an event
8	predict the probability of outcomes by specifying the probability of simple events or an impossible event
2	specify the probability of an impossible event
10	interpret that the measure of the likelihood of an event can be represented by a number from 0 to 1 by specifying the outcome of an event between 0 and 1
1	describe events as likely or unlikely by identifying the most likely event of two unequally likely events
6	find outcomes of probability experiments and decide if they are equally likely by identifying two equally likely events as being equally likely

Experimental Fraction Instrument

To measure incidental fraction learning, the researcher also administered a parallel Experimental Fraction Instrument (EFI) (see Appendix B) as a pretest and posttest. The role of the EFI was to measure students' incidental fraction learning skills after working with experimental probability.

To insure that the content of the instrument reflected content domain of elementary students regarding the concept of fractions, the researcher's teaching experience and the literature were used as a guide. The researcher incorporated items for the EFI similar to those found in elementary mathematics textbooks.

Pilot Study

The researcher conducted a pilot study during the Spring 2000 term using two fifth-grade elementary classes in a school district in the southeastern United States. The purpose of the pilot study was to obtain reliability data for the instruments and examine the application of the treatment.

After the implementation of the EPI and EFI in one class, the researcher made several changes regarding the content of the instrument. First, objects representative of manipulatives were redrawn so that the values were a true representation of the size. These objects included marbles and spinners. Second, more specific directions were given. Third, the researcher eliminated two questions because they were repetitive.

The researcher observed that the success of the EPI for students was probable only when students were given an opportunity to try experiments using the computer-simulated probability software. Because students were not familiar with some of the terminology used in the lessons, the researcher designed the lessons to provide time for whole group discussion prior to the activities for all groups in the study. This eliminated many questions except a few that related to the activities. For example, students discussed the term least and gave examples of least likely events. All groups continued in this way until all unfamiliar terms were understood.

An internal consistency reliability estimate for the final version of the EPI was estimated using an EXCEL program. The reliability estimate for the EPI was given as .89. An item analysis was done to determine if items had a reasonable level of discrimination. The difficulty and discrimination indices for the individual items of the EPI are reported in Table 4. Item discrimination for the EPI ranged from 0 to .26 with a median of .21.

An EXCEL program was also used to estimate the EFI. The reliability estimate for the EFI was also .89. The difficulty and discrimination indices for the individual items of the EFI are reported in Table 5. Item discrimination for the EFI ranged from 0 to .27 with a median of .15.

The discrimination index is used to describe the validity of a test in terms of the persons in contrasting groups who pass each item. The researcher obtained the discrimination index by subtracting the proportion of students in the lower half of the group who answered the item correct from the proportion of students in the upper half of the group who answered the item correct. The item-total correlation as a measure of discrimination for EPI and EFI was given as .55 and .32, respectively.

The item difficulty index is used to describe the percentage of persons who correctly answer a particular test item. The difficulty index was obtained by the number of subjects who answered the item correctly, divided by the total number of subjects taking the test.

The first administration of the study was conducted during September 2000. The study included four classes taught by the classroom teachers. The study began with the administration of the EPI and the EFI in all classes. After implementation of the two

Table 4

Difficulty and Discrimination Indices for Experimental Probability Instrument

Item	Discrimination	Difficulty	p-value
1	.250	.291	.56
2	.239	.242	.62
3	.247	.370	.42
4	.156	.434	.81
5	.234	.311	.64
6	.198	.121	.26
7	.234	.468	.36
8	.250	.375	.43
9	.247	.429	.58
10	.215	.477	.70
11	.207	.602	.72
12	.156	.546	.81
13	.229	.448	.66
14	.222	.448	.68
15	.250	.273	.57
16	.117	.162	.87
17	.253	.473	.45
18	.198	.373	.74
19	.198	.104	.74
20	.156	.439	.81
21	.179	.399	.77
22	.131	.297	.85
23	.215	.712	.70
24	.215	.217	.30
25	.144	.490	.83
26	.168	.431	.21
27	.168	.545	.21
28	.102	.406	.11
29	.131	.453	.16
30	.254	.430	.47
31	.244	.678	.60
32	.244	.291	.60
33	.215	.375	.30
34	.222	.561	.68
35	.234	.784	.64
36	.222	.801	.68
37	.222	.801	.68
38	.234	.559	.64
39	.198	.508	.74
40	.250	.340	.57
41	.244	.658	.40
42	.168	.090	.21
43	.179	.505	.23
44	.000	1	0
45	.000	1	0
46	.255	.550	.51

Table 5

Difficulty and Discrimination Indices for Experimental Fraction Instrument

Item	Discrimination	Difficulty	p-value
1	.152	.716	.83
2	.152	.716	.83
3	.152	.716	.83
4	.152	.716	.83
5	.083	.409	.92
6	.000	0	1
7	.083	.409	.92
8	.000	0	1
9	.083	.409	.92
10	.000	0	1
11	.083	.409	.92
12	.000	0	1
13	.000	0	1
14	.083	.409	.92
15	.000	0	1
16	.083	.409	.92
17	.152	.553	.83
18	.152	.770	.83
19	.205	.503	.75
20	.242	.444	.67
21	.242	.444	.67
22	.152	.552	.83
23	.273	.750	.50
24	.265	.373	.42
25	.205	.316	.75
26	.000	0	1
27	.152	.498	.83
28	.242	.315	.67

instruments, the researcher decided to make a change to the format of the instruments. The researcher attached the EFI onto the EPI to encourage students to complete the instruments.

The classroom teachers were instructed to teach the probability lessons to students following the administration of the covariates. Several changes were made to the lessons based on the researcher's observations. The lengths of the lessons for the manipulative and the computer/manipulative group were shortened to provide enough time for discussion. Manipulatives were presorted and labeled before the treatment lessons for affective use of time. All classes except the control group were assigned to small groups within the classes prior to the start of the lessons. Another major change based on the researcher observation was for the researcher to facilitate the lessons instead of training the teachers to facilitate the lessons. The reason was that although the teachers were trained and given materials to use for the lessons, they did not follow through with their roles in the study. One classroom teacher did not use any of the concrete manipulatives provided. Another classroom teacher did not follow the lessons, which were designed according to the constructivist theory. The teacher in this classroom taught using more traditional teaching methods. For example, the teacher would have the students conduct a simulation using marbles on the computer and then would ask how many of each did students obtain. There would be no further discussion of the experience. This continued for 5 minutes, and then the teacher concluded the lesson. Students were not able to discuss what they learned from their results of the simulations. The researcher concluded that the study could not be conducted properly without adherence to the theoretical framework.

Research Population and Sample

The population for this study consisted of students in the fifth grade at public elementary schools. The research sample consisted of 83 fifth-grade students enrolled in

elementary school in the southeastern United States. The subjects were of various abilities, mixed gender, and racial background. The students in the classes represented characteristics of the population of students in these elementary schools. There were four classes of students: three treatment classes and one control class. Because the students were already in their assigned classes, true random assignment was not possible. First, students in one class participated with concrete manipulatives. A second class experienced instruction with the computer and probability software. A third class was taught using a combination of concrete manipulatives and the computer. The control group participated in the traditional setting.

Procedures

Prior to the study, the researcher obtained permission from the University of Florida Institutional Review Board for the investigation to take place. The students were informed and had to obtain parental permission to participate in the study (see Appendix E). The classroom teachers administered the pretest and posttest instruments in class. A schedule for the procedures is provided in Table 6.

Table 6

Schedule of Administration of Instruments and Treatment

Group	Pretests	Treatment	Posttests
Treatment(c/m)*	January 8, 2001	January 9-11, 2001	January 12, 2001
Treatment(m)**	January 8, 2001	January 22-25, 2001	January 26, 2001
Treatment (c)***	January 8, 2001	January 22-25, 2001	January 26, 2001
Control	January 8, 2001	January 16-18 2001	January 19, 2001

** manipulative group; *computer & manipulative group; ***computer group

Treatment

Treatment consisted of the use of concrete manipulatives and computer-simulated manipulatives during in-class lessons. There were three preplanned lessons, each lasting one 60-minute class period. The lessons were presented in a constructivist perspective. According to information provided in the review of the literature, this included student discovery, collaboration with other students in groups, and doing activities that relied heavily on sources of data and manipulatives. Rather than correct students' initial analysis, the researcher let the experimental results guide and correct students' thinking (Van de Walle, 1998). This one instructional method gave students an opportunity to develop and build upon their own mathematical ideas. Rather than the teacher being the dispenser of knowledge, students were able to use their own inquiry to investigate events.

The concrete and computer-simulated manipulatives used in this study included coins, dice, spinners, and marbles. These manipulatives were used as simulations of real situations. These manipulatives can illustrate impossible and certain events. All of these manipulatives can be used to teach basic concepts of probability. Students need to see that simulations aid in learning fundamental concepts of probability.

Treatment Lessons

The researcher taught all classes. Each group had its own version of the lessons. This was due to the type of materials used for each group. Although the lessons were designed according to the nature of the group, the objectives for the lessons for each group were parallel. All treatment groups were put into small cooperative groups of four to six students. The lessons used during the treatment phase are found in Appendix C. A brief outline follows:

Lesson 1.

Objective: To have students explore concepts of chance and to decide whether a game is fair or unfair using concrete manipulatives or computer generated manipulatives.

Procedure: Students worked in cooperative groups of four and six. Students in each group had a role in their group. The roles in the group included two students in charge of the manipulatives, one student to record the outcomes and a student to report the information to the whole class on the performance of their group. The instructor assessed students' prior knowledge through inquiry about chance and luck. Also, through whole class discussion, students described what fair meant to them. The students spent the remaining class time working on activities and reflecting on their outcomes through group and whole class discussion. The activities are described in detail in the motivational sections of the lessons in Appendix C.

Lesson 2

Objective: To have students work with likely or unlikely events.

Procedure: Students worked in cooperative groups. The instructor checked for understanding of terms by first having students discuss in their groups what the terms certain, impossible, likely, or unlikely meant to them. In whole class setting, students discussed the meaning of these terms. Students worked on activities in cooperative groups. Following the activities, students responded to questions regarding their outcomes.

Lesson 3

Objective: To have students make and test predictions for simple experiments.

Procedure: Students spent the first part of class time working on activities in their cooperative groups and remaining class time reviewing concepts of chance, and simple, likely and unlikely events with the instructor.

According to the NCTM (2000), the study of chance done with the use of concrete manipulatives should be followed by computer-simulated manipulatives using available software. This would gradually lead to a deeper understanding of these ideas through middle and high school. Therefore, the researcher chose to use a computer software program, "Probability Explorer," developed by Drier (2000). Drier designed the program to generate simulations of manipulatives for large trials in a few minutes. The software displays results as a table that presents the experimental results as a frequency (count) or relative frequency (fraction, decimal, or percent), a pie graph that displays the relative frequencies, or as a bar graph that shows the frequency distribution. These added features of this newly designed software program give teachers an opportunity to reinforce previously taught skills. According to many researchers (e.g., Weibe, 1988; Perl, 1990; Strommen & Lincoln, 1992; Clements, 1998; Greening, 1998), students' experiences with technology provide deeper and more substantial understandings.

Data Analysis

A nonequivalent control group design was used for the study. There were four groups with one class of students in each. Treatment Group I was allowed to use concrete manipulatives for the lessons. Treatment Group II participated in computer simulations with the lessons, and Group III used a combination of both concrete manipulatives and computer simulations for the lessons. The Control Group was not

provided with manipulatives or the use of the computer for use with the lessons. The quasi-experimental research design involved a 2x2 matrix: the level of concrete manipulative use and the level of computer simulation. The objective was to determine the effect of the two independent variables, individually and interactively, on the dependent variable (posttest scores). The researcher used an analysis of covariance (ANCOVA). The advantage of using an ANCOVA was to control for any initial differences that may have existed between the four groups and to determine if there was a significant treatment effect. The EPI and the EFI were used as pretests (covariates) and as posttests.

In addition to the quantitative analyses of the instrument, the teachers in the study classes observed the experimental classes to record students' reaction to the use of concrete manipulatives and computer-simulated manipulatives. These observations were used to examine students' problem-solving process.

CHAPTER 4
RESULTS

This chapter contains the descriptive statistics and the results of the analysis pertinent to the null hypotheses of this study. Participants for the study consisted of 83 fifth-grade students from four classes at three elementary schools in Southeast U.S. The instructional phase consisted of three lessons that tested students understanding, learning skills, concepts and ability of experimental probability.

Descriptive statistics for the pretest and posttest results for the probability instrument are presented in Table 7, which includes the number (n) in each group, mean (m), and standard deviation (SD).

Table 7

Pretest and Posttest Means and Standard Deviations for EPI

Group	n	Pretest Mean %	Pretest SD	Posttest Mean %	Posttest SD
Computer & Manipulative	20	24.20	4.88	26.30	3.49
Computer	17	17.71	6.99	22.88	4.62
Manipulative	12	19.17	5.79	20.17	8.22
Control	17	25.47	6.52	26.09	7.67

The researcher employed an ANCOVA to control for initial differences that may have existed in the pretest results among the four groups. The ANCOVA results revealed

that when considering the dependent variable, posttest scores on the Probability Instrument, there were no statistically significant difference among students using concrete manipulatives based on the results of the EPI (see Table 8). Therefore, the following null hypotheses could not be rejected:

> There will be no significant difference between students who use concrete manipulatives and students who do not use concrete manipulatives regarding students' learning skills and concepts in experimental probability.
>
> There will be no interaction between students who use computer-simulated manipulatives and concrete manipulatives regarding students' learning skills and concepts in experimental probability.

These findings indicated that the interaction terms did not account for a significant proportion of the variance in posttest scores (adjusted $R^2 = .3861$).

Table 8

Analysis of Covariance: EPI

Source	DF	Type III SS	F	p
Pretest	1	374.21	15.07*	0.00
Manipulative	1	14.71	0.59	0.44
Computer	1	134.27	5.41*	0.02
Computer- Manipulative	1	29.28	1.18	0.28
Pretest x Manipulative	1	12.22	0.49	0.49
Pretest x Computer	1	152.90	6.16*	0.02
Pretest x Computer/Manipulative	1	23.54	0.95	0.33
Model	7	1189.44	6.841*	0.00
Error	58	1440.68		

Note: *significant for p= .05

After these terms were removed, there was minimum change in the observed variance (adjusted $R^2 = .3833$). However, there was a statistically significant variable for students' use of computer-simulated manipulatives on the EPI (see Table 8). Therefore, the following null hypotheses could be rejected:

> There will be no significant difference between students who use computer-simulated manipulatives and students who do not use computer simulated manipulatives regarding students' experimental probability learning skills and concepts.

The parameter estimate (see Table 9) indicates how the treatment groups faired compared to the control group. Results of the interaction analyses show that achievement in mathematics can significantly interact with computer instruction. Students with very low scores on the EPI received higher scores on the EPI when instruction included computer instruction. On average, students in the computer group scored 13 points higher on the posttest than students in the control group given their individual pretest scores. This was possible only when students in the computer group had low pretest

Table 9

EPI: Comparison of the Parameter Estimate for the Treatment and Control Groups

Source	Estimate	p
Manipulative	-5.55	0.44
Computer	14.06*	0.02
Manipulative-Computer	8.30	0.28
Pretest x Manipulative	0.22	0.49
Pretest x Computer	-0.64*	0.01
Pretest x Manipulative-Computer	-0.29	0.33

Note: *significant for $p = .05$

scores on the measure of probability skills. However, this advantage disappears when comparing students who have higher pretest scores (see Figure 1). Low-achieving students were obviously not benefiting as much from the usual mode of instruction employed by their teachers, and even though a computer approach was new to them, it proved to be more beneficial than the more familiar type of instruction. This indicates that students who have little previous probability knowledge would benefit from participation in the computer group (see Figure 1). It is also an indication that computer use shows positive results as indicated by Connell (1998), Kim (1993), and Terry (1995).

In addition, the same interpretation applies when comparing the computer group to the manipulative group and the manipulative-computer group (see Figures 2 and 3).

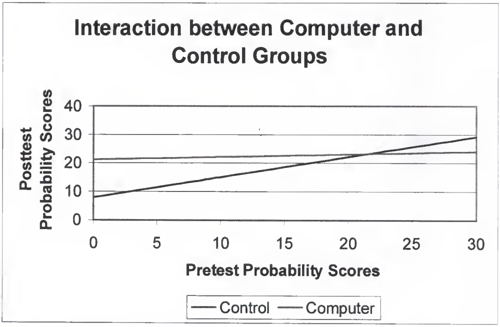


Figure 1. Interaction between computer and control groups.

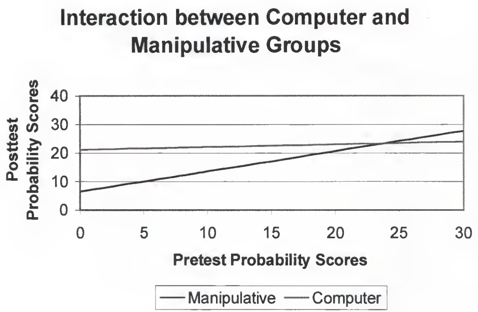


Figure 2. Interaction between computer and manipulative groups.

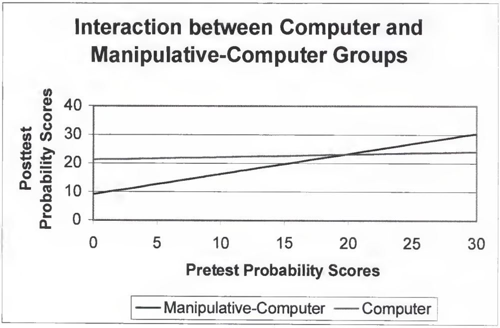


Figure 3. Interaction between computer and manipulative-computer groups.

Students in the computer group who have low pretest scores on probability skills tend to do better than students in the manipulative group and the manipulative-computer group. This is an indication that neither the manipulative group, nor the manipulative-computer group was significantly different from the control group. However, like the control group, the manipulative group and the manipulative-computer group were similarly different from the computer group.

Other Findings

Incidental learning, which is not planned, is often overlooked. The study of probability gives students an opportunity to revisit and practice previously learned concepts and skills. Therefore, the researcher included an EFI to measure incidental learning of fractions. Descriptive statistics for the pretest and posttest results for the EFI are presented in Table 10, which includes the number (n) in each group, mean (m), and standard deviation (SD).

Table 10

Pretest and Posttest Means and Standard Deviations for EFI

Group	n	Pretest Mean %	Pretest SD	Posttest Mean %	Posttest SD
Control	16	26.13	4.05	26.94	3.92
Manipulative	12	18.17	7.57	19.75	8.17
Computer	16	14.56	9.64	22.63	5.82
Computer & Manipulative	20	20.25	5.93	22.05	4.84

Significance tests of the parameter estimates for group affiliation indicate that there was no detectable difference between the mean adjusted posttest scores of students in the control group and the mean adjusted posttest scores of students in the computergroup or the computer-manipulative group using a significance level of $p = 0.05$ given the students fraction pretest scores (see Table 11). The only significant difference was in the scores of the students in the manipulative group (see Table 11). Even with the interaction terms removed, there was still a relatively significant change in the scores of the students in the manipulative group (see Table 12). Controlling for fraction pretest scores, students in the manipulative group still scored on average, 3.30 points lower on the fraction posttest than students in the control group (see Table 12). Students in the computer group and the

Table 11

Analysis of Covariance: Fraction Instrument

Source	DF	Type III SS	p	Parameter Estimate	F
Model	7	1222.13	0.00		8.96
Error	56	1090.86			
Fraction Pretest	1	51.16	0.11	0.46	2.63
Manipulative	1	34.96	0.19	-10.98	1.79
Computer	1	3.86	0.66	3.43	0.20
Manipulative-Computer	1	4.09	0.65	-3.79	0.21
Pretest x Manipulative	1	29.49	0.22	0.41	1.51
Pretest x Computer	1	6.02	0.58	-0.17	0.31
Pretest x Manipulative-Computer	1	1.09	0.81	0.08	0.06

Table 12

Analysis of Covariance: Fraction Instrument

Source	DF	Type III SS	Parameter Estimate	p-value	F
Model	4	1074.63		0.00	12.80
Error	59	1238.35			
Fraction Pretest	1	679.53	0.48	0.00	32.38
Manipulative	1	67.42	-3.36	0.06	3.21
Computer	1	9.15	1.25	0.18	0.44
Manipulative-Computer	1	34.21	-2.06	0.17	1.63

manipulative-computer group still did not appear to have different posttest scores than students in the Control group.

Not all unplanned learning is effective. Incidental learning is generally not recognized or labeled as learning by the learner or others; therefore, it is difficult to measure.

Limitations of the Study

Several limitations must be considered when interpreting the results of this study. Due to the time of year, the classes were intact and nonrandomly assigned to treatment and control groups. This is not uncommon among studies conducted in school settings. There may have been preexisting differences in the intact classes to begin with. For example, the computer/concrete manipulative group was a higher scoring class to begin with. Because of intact classes, it was not possible to take several classes of students and

include them in each group. Therefore, only one class of students, which included 16-25 students, was involved in each group. This is the reason for the small sample size.

The Probability Explorer was new software that had never been used in whole class instruction. Therefore, the researcher was unable to compare the results to that of students who used the Probability Explorer software in whole class instruction. In addition, the software was only designed for use in IBM computers, which limited where it could be used in the school district.

The duration of the study was only 4 days. The question might be raised whether 4 days is sufficient time to develop the concepts completely and effectively with computers and concrete manipulatives. The researcher believes that because traditional teaching practices are still used in many classrooms, students may not be used to a constructivist learning environment where students are actively engaged in their learning. Thus, it may take students a few trials to become adjusted to different teaching and learning styles. However, because of the schools curriculum, school administrator and teachers limited outside activities to a few days.

Finally, the researcher has been a mathematics teacher in the southeastern United States for more than 10 years and was aware of the possibility that the researcher influencing results may introduce a threat to validity. Using specific preplanned lessons for students in all the study classes minimized this.

CHAPTER 5
CONCLUSION

Summary

The purpose of this study was to examine the effectiveness of concrete and computer-simulated manipulatives on elementary students' probability learning skills and concepts. There were four heterogeneous groups with one class of students in each. Treatment Group I was allowed to use concrete manipulatives for the lessons; Treatment Group II participated in computer simulations for the lessons; and Group III participated in a combination of both concrete manipulatives and computer simulations for the lessons. The control group was taught the lessons using traditional means of instruction. The objective was to determine the effect of computer-simulated manipulatives and concrete manipulatives on elementary students' probability learning skills and concepts on the dependent variable of posttest scores on the EPI.

The sample consisted of 83 fifth-grade students. The classes remained intact; hence, random assignment to treatment groups was not possible. The researcher taught all classes. Following a 2-week break, students in both the treatment and control groups were administered the EPI and the EFI as pretests. Approximately 2 weeks elapsed between the first and second testing sessions during which time the instruction took place.

An ANCOVA was used to examine the posttest scores from the EPI. Based on the ANCOVA, it was concluded that students experiencing computer instruction, who

had low pretest scores, significantly outperformed students experiencing traditional teaching instructions on content measures of the concepts of probability. Thus, the ANCOVA results led to the rejection of the following null hypothesis:

> There will be no significant difference between students who use computer-simulated manipulatives and students who do not use computer simulated manipulatives regarding students' experimental probability learning skills and concepts.

The following null hypotheses could not be rejected:

> There will be no significant difference between students who use concrete manipulatives and students who do not use concrete manipulatives regarding students' experimental probability learning skills and concepts

> There will be no interaction between students who use computer-simulated manipulatives and concrete manipulatives regarding students' experimental probability learning skills and concepts.

Unlike Fischbein and others (1991) where the researchers indicated that students had difficulty answering probability problems because of their lack of skills for rational structures, the analysis of the EFI indicated that controlling for fraction pretest scores did not reveal significant differences on the fraction posttest scores for students in the computer group, manipulative group, and the computer/manipulative group when compared to the control group. Like Fischbein and others (1991), it may be suggested from the results that the reason some of the groups did not have significant gains on the probability posttest when compared to the control group was because of their lack of skills for rational structures.

Discussion

Although the means for the treatment and control groups would indicate that the students were not successful with the EPI, the computer group only showed significant

gains as measured on the EPI. The instrument was designed to measure the students' learning skills and concepts of experimental probability. Interaction with the use of computer simulations was found to significantly affect students' concept of probability. Further analyses of the interaction, as in Jones and others (1999), revealed that students in the computer simulations group only who began instruction below an informal quantitative level were more affected by the treatment. Jiang and Potter (1994), who also investigated students' conceptual understanding of probability, found that computer simulations were useful for above average students but were not sure of the benefits for students below average. However, the students in the current study who were in the computer group only and who were below average to begin with showed significant increases on their understanding of probability concepts.

Students in the computer group and the computer-manipulative group had never seen simulations with dice, coins, and marbles performed on the computer before this study. The probability Explorer program developed by Drier (2000) was used for the first time in whole class learning and group instruction by the researcher. Thus, computer simulation use in the classes required less time to manipulate. Choi and Gennaro (1987), and Clements and McMillan (1996) contended that the use of computer simulated materials allows students to achieve more in less time than the required time for concrete manipulative experiences. Hence, students in the concrete-manipulative group and the concrete and computer simulation group showed no significant differences in their learning of probability concepts when compared to the control group. This may be attributed to the fact that the use of concrete manipulatives requires more time, and once

a series of actions is finished, it is often difficult to reflect on it (Clements & McMillan, 1996).

The results of the EPI indicated that some areas of concern still exist. Students' lack of understanding was identified in probability of events and probability comparisons as follows (a) predicting most or least likely events based on subjective and quantitative judgments, (b) predicting most or least likely events for single stage experiments, (c) using numbers informally to compare probabilities, and (d) making probability comparisons based on quantitative judgments. Jones and others (1999) also identified these as problem areas for students.

Implications

There is a need to include fraction concepts when studying probability to help students see the critical role fraction concepts play in the study of probability. However, there is no reason students should postpone studying concepts of probability after fraction skills have been mastered. The study of probability could possibly serve as an introduction to fractions. It may be that a spiral curriculum, designed to increase student interest and motivation in a particular subject area, would be useful for students when studying probability. With the repetition of fundamentals and the integration of subject content, the spiral curriculum is expected to increase retention of basic skills and concepts.

This study has added further to the impact that technology is making on education. Students with low scores on probability concepts seemed to benefit more from the computer instruction than any of the other groups. However, the use of computer instruction only is not sufficient to insure students' construction of probability

concepts. Having appropriate software available, like the Probability Explorer, is important. In addition, teacher training on the use of the software and being able to provide students with a constructivist learning environment, which emphasizes understanding and builds on students' thinking, is necessary to help students develop to their full potential in mathematics education.

Recommendations

This study has demonstrated that probability is a very useful concept that needs to be started early in the development of children and continuously revisited. Several suggestions for future research have developed as a result of this study. How can incidental learning be measured when studying probability? Future researchers should investigate how studying probability at the early elementary level will influence students' understanding of these concepts and achievement in other areas of mathematics.

Is it necessary to study fractions prior to studying probability? Longer studies over time are needed to determine if studying probability increases students understanding of fractions. Future studies are also needed to determine if students with little fraction knowledge benefit more from studying probability before the introduction of fraction concepts or after. This study revealed that students with little prior fraction knowledge benefited when using the Probability Explorer. However, future studies are needed to determine the impact of the Probability Explorer and other available probability software on teaching and learning of probability concepts over an extended period. Does probability software helps students with low fraction skills and confuses students with prior fraction skills? If probability software helps students with little prior fraction skills, then studying probability concepts early in children's developmental stages

is warranted. Some questions to ask are the following: Does using probability software help students understand terms such as certain, likely, or least? Does using the probability software help students exercise higher order thinking skills? How is probability software helpful for teaching and learning in other areas of mathematics? After instruction with probability software, do teachers teach more effectively than teachers not using this type of instruction? Does using probability software make it easier to introduce probability at the elementary level? Research needs to be conducted on available material useful for learning probability on the computer that would facilitate appropriate learning environments that would further develop the understanding of children's experimental probabilistic reasoning skills.

Furthermore, there is a continuous need to determine how concrete manipulatives compared to computer-simulated manipulatives affect students' learning of concepts. According to the observations from the teachers in the study, students were less convinced of the likelihood of an event happening when they used computer-simulated manipulatives compared to concrete manipulatives. Although students observed that from large probabilities, chances appeared more equal, they were more comfortable with the results of the outcomes when they were able to touch and feel the objects. Some students posited that the computer had a set number of patterns that it would produce each time.

APPENDIX A
EXPERIMENTAL PROBABILITY INSTRUMENT

Student Name________________________

Read and answer each question.

1. A fair coin is flipped four times, each time landing with heads up. What is the most likely outcome if the coin is flipped a fifth time?

    a. Another heads is more likely than a tails.
    b. A tails is more likely than another heads.
    c. The outcomes (head and tails) are equally likely.

2. Which of the following is the most likely result of five flips of a fair coin?

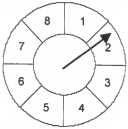
    a. HHHTT
    b. THHTH
    c. THTTT
    d. HTHTH
    e. All four sequences are equally likely.

3. Which of the following sequences is the least likely to result from flipping a fair coin five times?

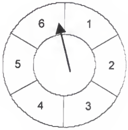
    a. HHHTT
    b. THHTH
    c. THTTT
    d. HTHTH
    e. All four sequences are equally unlikely.

4. Spinner A and Spinner B are used to play this game.
   You win if the arrow stops on 5. You lose if the arrow stops on any other number.
   (If the arrow stops on a line, you spin again)
   If you play this game only once, which spinner would you choose so that you would have the better chance of winning?

Spinner A



Spinner B

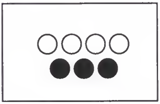


    a. Spinner A
    b. Spinner B
    c. It does not make any difference.

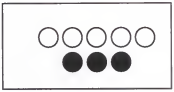
Box A and Box B below are used to play this game.

5. To play this game you pick a chip from one of the boxes without looking.
   You win if you pick a black chip.
   You lose if you pick a white chip.
   If you play this game only once, which box would you choose to pick from so that you would have the better chance of winning?

Box A



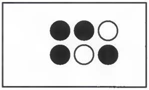
Box B



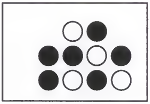
a. Box A
b. Box B
c. It does not make any difference

6. In the Box C and Box D below;
   You win if you pick a white chip.
   You lose if you pick a black chip.
   If you play this game only once, which box would you choose to pick from so that you would have the better chance of winning?

Box C



Box D

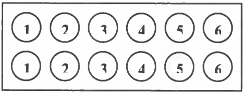


a. Box C
b. Box D
c. It does not make any difference.

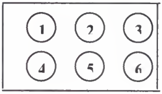
Box E and Box F below are used to play this game.

7. To play this game you pick a chip from one of the boxes without looking.
   You win if you pick a chip with a 3 on it.
   You lose if you pick a chip with any other number on it.
   If you play this game only once, which box would you choose to pick from so that you would have the better chance of winning?

Box E

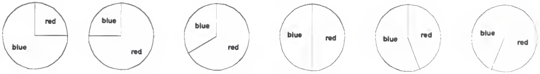


Box F



   a. Box E
   b. Box F
   c. It does not make any difference

Use the pictures below for questions 8 and 9.



A B C D E F

8. Which letter of the spinners whose pointer is more likely to stop on red than blue.

   a. A or B
   b. C or D
   c. E or F
   d. B or C
   e. D or E
   f. C or F

9. Look at Spinner D and answer the following questions:

   a. Could you get 100 reds in 100 spins on this spinner? __________

   b. Are you likely to get 100 reds in 100 spins on this spinner? ______________

   c. About how many reds do you expect from 100 spins? ____________________

Questions 10-18 are statements about chance events. For questions 10-18, if you think a statement is true put a T in the blank after the statement. If you think the statement is not true put an F in the blank.

10. If a two-sided (head/tails) coin does not stand on its edge after it is flipped, it is certain to show either heads or tails. _______

11. If we toss a fair coin once, we are as likely to get a head as a tail. ________

12. If we toss a fair coin 100 times, it may be heads 0 times, or 100 times, or anything in between. ________

13. If we toss a fair coin 1000 times, it is very unlikely that we will get 900 tails. _______

14. Whether we get heads or tails when we toss a fair coin is a matter of chance. _______

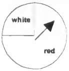
15. You might toss a fair coin 1000 times without getting a single head. _______

16. If a box contains two blue marbles and one red one, and you pick one marble without looking, the chances are 2 out of 3 that it will be blue. _______

17. In problem number 16, you have one chance in three of picking a red marble. _______

18. In problem number 16, your chances of picking a green marble are zero. _______

A spinner has a dial that is one-fourth ($\frac{1}{4}$) white and three-fourths ($\frac{3}{4}$) red. Use this information to answer questions 19-23.



19. If you spin the pointer 10 times, are you likely to get the same number of reds as whites? yes or no? _____________

20. Are you likely to get more whites than reds? yes or no? ___________

21. If the chances of getting red are 3 out of 4, what are the chances of getting white? _____ out of _______

22. Can you be certain of getting at least one red in 10 spins? yes or no? _________

23. Is it very likely that you will get no reds in 10 spins? yes or no ______

Read the following statement carefully and then answer questions 24-28.

James has three green marbles and two blue marbles in his pocket.

24. How many marbles must he remove to be sure of getting a blue marble?

    ____

25. How many marbles must he remove to be sure of getting both the blue ones?

    ____

26. How many marbles must be removed to be sure of getting both colors?

    ____

27. How many marbles must be removed to be sure of getting a green one?

    ____

28. If James removes one marble, there are three chances out of _______ that it will be a green one.

29. If the dial of a spinner is all red, we say the chance of red is equal to:

    a. any other chance
    b. one chance in two
    c. one-half
    d. one

30. If the dial of a spinner is all blue, we say the chance of red is equal to:

    a. one
    b. zero
    c. one chance in one
    d. one-half

31. A spinner has a dial, which is evenly divided, into red, white, and blue spaces. Write a number sentence that describes the chance of getting blue.

    ____

32. A bag contains several marbles. Some are red, some white, and the rest blue. If you pick one marble without looking, the probability of red is $\frac{1}{3}$ and the probability of white is $\frac{1}{3}$. What is the probability of blue? __________

33. A bag contains one red marble, two white marbles, and three blue marbles. If you pick one marble without looking, what is the probability that the marble will be red? __________

34. In problem 33, what is the probability that the marble will be white? __________

35. In problem 33, what is the probability that the marble will be blue? __________

36. A bag contains ten marbles, 6 are white and 4 are blue. If one is drawn out without looking, what is the chance that the marble drawn out will be white?

    a. 1 out of 10
    b. 4 out of 10
    c. 6 out of 10
    d. None of these

37. A bag contains 4 white marbles, 2 blue marbles, and 6 yellow marbles. A blindfolded person draws out one marble. What is the chance that the marble will be either white or blue?

    a. 2 out of 12
    b. 4 out of 12
    c. 6 out of 12
    d. None of these

38. Suppose you flipped a coin 100 times and got 47 heads and 53 tails. Would you say:

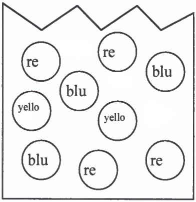
    a. The coin is unfair in favor of Heads.
    b. The coin is unfair in favor of Tails.
    c. The coin is fair.
    d. I cannot tell if the coin is fair or unfair. Why or why not? ______________

39. What is the probability of rolling an even number on a fair die, numbered 1-6?
    ___ out of ____

40. Jerry rolled a fair six-sided die ten times and got 8 odd numbers and 2 even numbers. How many odd numbers do you think he will have rolled after 100 tries?

    a. 0-15 odd numbers
    b. 15-25 odd numbers
    c. 25-35 odd numbers
    d. 35-45 odd numbers
    e. 45-55 odd numbers
    f. 55-65 odd numbers
    g. 65-75 odd numbers
    h. 75-85 odd numbers
    i. 85-100 odd numbers

41. If you draw a marble out of the bag below without looking, which color do you have the least chance of picking?____________



42. If you draw a marble out of the same bag without looking, which color do you have the best chance of picking? ___________

APPENDIX B
EXPERIMENTAL FRACTION INSTRUMENT

Student Name________________________

1. What is 7 of the fraction $\frac{7}{10}$ called? ______________________________

2. What is 6 of the fraction $\frac{2}{6}$ called? ______________________________

3. What is the denominator of the fraction $\frac{1}{3}$? __________________________

4. Which is the larger number, the numerator or the denominator of $\frac{3}{4}$? __________

5. Write the fractions that mean the same as:

   a. one-half _____ b. two-thirds ______ c. four-eighths ______

   c. one of three parts _____ d. one of four parts ______

Add or subtract the fractions:

6. $\begin{array}{r} \frac{3}{6} \\ - \quad \frac{2}{6} \\ \hline \end{array}$

7. $\frac{1}{6} + \frac{3}{6} + \frac{2}{6} =$ ________

8. $\frac{3}{7} + \frac{2}{7} =$ ________

9. $\frac{1}{3} + \frac{1}{3} =$ ________

10. $\frac{3}{8} + \frac{5}{8} =$ ________

11. $\frac{3}{5} + \frac{1}{5} =$ ________

Box A

Math Quiz			
1. d	√	6. c	X
2. a	X	7. i	√
3. a	√	8. k	√
4. f	√	9. g	√
5. h	X	10. j	√

Use Box A to answer questions 12-14.

12. There are ____ answers on the paper.

13. _____ of the ____ answers are correct.

14. What fraction of the answers are correct? _____

Box B

b	D	A
T		R
i	N	g
u	m	B

Use Box B to answer questions 15-17.

15. There are _____ letters in the group.

16. ____ of the _____ letters are capitals.

17. What fraction of the letters are capitals? _____

Box C

graph	add	circle	counting	cube	decimal	divide	divisor
zero	line	meter	number	plane	point	set	square

Use Box C above to answer questions 18-21.

18. What fraction of the words begin with **p**? ________

19. What fraction of the words end with **e**? ________

20. What fraction of the words have the letter **t**? ________

21. What fraction of the words have exactly four letters? ________

Write the fractions in words just as you would say them aloud.

22. $\frac{1}{2}$ ________________________________

23. $\frac{2}{3}$ ________________________________

24. Circle all the fractions equal to one.

$\frac{11}{111}$ $\frac{15}{23}$ $\frac{5}{6}$ $\frac{18}{18}$ $\frac{3}{10}$ $\frac{1}{6}$ $\frac{100}{100}$ $\frac{5}{12}$ $\frac{6}{101}$ $\frac{6}{6}$ $\frac{1}{12}$ $\frac{3}{4}$ $\frac{2}{2}$ $\frac{7}{8}$ $\frac{1}{1}$

APPENDIX C
LESSONS

Lesson 1 *control group*

Goal: Develop conceptual understanding of chance.

Objective:
The students will

- Explore concepts of chance.
- Decide whether a game is fair or unfair

Materials:

- paper and pencil to record information

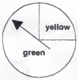
Procedure:

- Teacher will write information on the board.

Motivational Activity 1:

- Call on students to describe different games or activities they know are determined by chance or luck.
- Ask students what is meant by a fair game.

Motivational Activity 2:



Draw a spinner on the board like the one to the right.

Students will look at the spinner, read each statement, and decide if the game is fair or unfair. Students will record their answers.

Game 1:

Team A wins 1 point if the pointer stops on green. Team B wins 1 point if it stops on yellow.______________

Game 2:

Team A wins 1 point if the pointer stops on green. Team B wins 2 points if it stops on yellow.______________

Game 3:

Team A wins 2 points if the spinner stops on green. Team B wins 4 points if it stops on yellow.______________

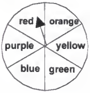
Game 4:

Team A wins 4 points if the spinner stops on green. Team B wins 6 points if it stops on yellow.______________

Lesson 1 continued, pg. 2

Motivational Activity 3:

Draw an equally spaced spinner on the board like the one below.



Students will look at the spinner, read each statement and decide if the game is fair or unfair. Have students record their answer.

Game 1:

Team A wins 1 point if the spinner lands on red, yellow, or blue. Team B wins 1 point if the spinner lands on orange, purple, or green.

____

Game 2:

Team A wins 1 point if the spinner lands on red or orange. Team B wins 1 point if the spinner lands on blue or green. No team gets any points if the spinner lands on purple or yellow. ______________

Game 3:

Team A wins 3 points if the spinner lands on red, orange, or yellow. Team B wins 4 points if the spinner lands on purple, blue, or green.

____

Game 4:

Team A wins 5 points if the spinner lands on purple. Team B wins 1 point if the spinner lands on red, orange, yellow, green, or blue.

____

Game 5:

Team A wins 1 point if the spinner lands on blue and 2 points if the spinner lands on green. Team B wins 2 points if the spinner lands on red, orange, or yellow. No team gets any points if the spinner lands on purple.

____

Game 6:

Team A wins 1 point if the spinner lands on purple or blue and 2 points if the spinner lands on red. Team B wins 2 points if the spinner lands on orange or yellow and 1 point if the spinner lands on green.

____

LESSON 2

Goal: Develop an understanding of likely or unlikely events

Objective:
The Students will:

- Describe events as likely or unlikely
- Discuss the degree of likelihood using such words as certain, equally likely, and impossible.
- Find outcomes of probability experiments and decide if they are equally likely or not.
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

Materials:
Paper and pencil

Motivational Activity 1:
In whole class discussion the following terms will be discussed to check for understanding: certain, impossible, or likely(maybe)
Judge whether the following events are certain, impossible, or likely (maybe):

- It will rain tomorrow
- Drop a rock in water, and it will sink
- Trees will talk to us in the afternoon
- The sun will rise tomorrow morning
- Three students will be absent tomorrow
- George will go to bed before 8:30 tonight
- You will have two birthdays this year

Motivational Activity 2:

Use the spinner to the right to answer question 1.

1. Which describes your chance of spinning a 3?

   (a) 1 chance in 100 (b) 1 chance in 10
   (c) 1 chance in 4 (d) 1 chance in 2

For questions 2-5, look at the spinner above in 1 to write a number sentence that describes your chances.

2. What are your chances of spinning the number 5?
3. What are your chances of spinning a number less than 5?
4. What are your chances of spinning the number 3?
5. What are your chances of not spinning a 2?

Lesson 2 continued, pg. 2

6. Have each student write down events on paper of a 6-sided number cube whose probabilities equal:

a. 0

b. $\frac{1}{6}$

c. $\frac{2}{6}$

d. $\frac{3}{6}$

e. 1

LESSON 3

Goal: Develop an understanding of simple events

Objective:
The students will:

- Predict the probability of outcomes of simple experiments

Materials:
Paper and pencil

Motivational Activity
Have students answer questions 1-5 and record their answers.

1. *Imagine in a drawer, there are 4 blue socks, 6 brown socks, and 8 black socks:*
   (a) How many socks would you have to pick before you could be sure of having picked one matching pair?
   (b) How many socks would you have to pick to get one pair of black socks?

2. *Suppose you are blindfolded and a friend places two different pairs of shoes in front of you and tells you to choose one pair:*
   What is the probability that the first two shoes you pick up will be a matched pair? Why?

3. *Imagine that you have 5 red cubes, 4 yellow cubes, and 3 green cubes in a bag,*

   (a) What is the probability of getting a yellow cube on the first draw? ______
   (b) What is the probability of getting a green cube on the first draw? _______
   (c) What is the probability of not getting a green cube on the first draw? ____
   (d) If you know that the probability of getting a yellow cube is 0, what does that tell you about the colors of the cubes in the bag? _______

4. *Imagine that you flipped a coin 25 times,*
   What do you think is the probability of getting heads?

5. *Imagine you flipped a coin 50 times,*
   Do you think the probability of getting a head is the same as your answer in 4? Why or why not?

Lesson 3 continued, pg. 2

Use the spinner below to answer questions 6 and 7.

6. If you spin a spinner like the one above, just one time, what are the chances of landing on 6?

7. What are the chances of landing on 3?

Lesson 1 *computer group*

Goal: Develop conceptual understanding of chance.

Objective:
The students will
- Explore concepts of chance through activities with spinners.
- Decide whether a game is fair or unfair

Materials:
Per cooperative learning group:
- Dice (generated on the computer)
- paper and pencil to record score

Per Class:
- chart paper (transparency and overhead markers)

Procedures:

Students work in cooperative groups ***(4-groups of four & 1-group of six)*** **(2-spinners; 1-recorder; 1-reporter)**
Each group will divide into two teams for each of the three motivational activities

Motivational Introduction:
- Call on students to describe different games or activities they know are determined by chance or luck.
- Ask students what is meant by a fair game.
- After the activity have students discuss whether the spinners used were fair or unfair.

Motivational Activity 1:
- For this activity, students will use a die generated from the computer software provided.
- Each of the 5-groups will divide into two teams (***team A & team B***).
- All the students on team A will be odd numbers (1,3, 5) and all the students on team B will be even numbers (2, 4, 6).
- The activity will start with one die generated on the computer.
- One person from each team (or the teacher) will take turns throwing the die on the computer.
- A team gets a point if any of their numbers come up on the die.
- The first team to get 15 points is the winner.
- This time the game will be played using two dice.
- One team will be even sum, and the other team will be odd sum.
- If the sum is odd, one team gets a point, if it's an even sum, the other team gets a point. The first team to get 10 points is the winner.

Lesson 1 continued, pg. 2:

Motivational Activity 2:

For games 1-4, each group will divide into two teams *(A & B)*. Use the computer-generated die. Each team will throw one die 4 times for each game. The team with the most points wins. Decide whether each game is fair or unfair. Have students record their results.

Game 1:

Team A wins 1 point if the die lands on 1, 2, 3, or 4. Team B wins 1 point if it lands on 5 or 6.

Game 2:

Team A wins 1 point if the die lands on 1, 2, 3, or 4. Team B wins 2 points if it lands on 5, or 6.

Game 3:

Team A wins 2 points if the die lands on 1, 2, 3, or 4. Team B wins 4 points if it lands on 5, or 6.

Game 4:

Team A wins 4 points if the die lands on 1, 2, 3, or 4. Team B wins 6 points if it lands on 5, or 6.

Motivational Activity 3:

Use a single die on the computer for this activity. For games 1-6, each group will divide into two teams *(A & B)*. Each team will throw one die 3 times for each game. The team with the most points wins. Decide whether each game is fair or unfair. Have students record their results.

Game 1:

Team A wins 1 point if the die lands on 2, 4, or 6. Team B wins 1 point if the die lands on 1, 3, or 5.

Game 2:

Team A wins 1 point if the die lands on 1 or 2. Team B wins 1 point if the die lands on 5 or 6. No team gets any points if the die lands on 3 or 4.

Game 3:

Team A wins 3 points if the die lands on 1, 2, or 4. Team B wins 4 points if the die lands on 3, 5, or 6.

Game 4:

Team A wins 5 points if the die lands on 3. Team B wins 1 point if the die lands on 1, 2, 4, 5, or 6.

Lesson 1 continued, pg. 3:

Game 5:

Team A wins 1 point if the die lands on 6 and 2 points if the die lands on 5. Team B wins 2 points if the die lands on 1, 2, or 4. No team gets any points if the die lands on 3.

Game 6:

Team A wins 1 point if the die lands on 3 or 6 and 2 points if the die lands on 5. Team B wins 2 points if the die lands on 1 or 4 and 1 point if the die lands on 5.

LESSON 2

Goal: Develop an understanding of likely or unlikely events

Objective:
The Students will:

- Describe events as likely or unlikely
- Discuss the degree of likelihood using such words as certain, equally likely, and impossible.
- Find outcomes of probability experiments and decide if they are equally likely or not.
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

Materials:

Coins
Number cubes or dice

Motivational Activity 1:
Each group will discuss the following terms with each other to check for understanding: certain, impossible, or likely(maybe)
Judge whether the following events are certain, impossible, or likely (maybe):

- It will rain tomorrow
- Drop a rock in water, and it will sink
- Trees will talk to us in the afternoon
- The sun will rise tomorrow morning
- Three students will be absent tomorrow
- George will go to bed before 8:30 tonight
- You will have two birthdays this year

Motivational Activity 2(students will work in their groups)

1. Have one student from each group come up and generate a coin toss on the computer
    - Group 1--(have them plug in 20 times)
    - Group 2--(have them plug in 25 times)
    - Group 3--(have them plug in 6 times)
    - Group 4--(have them plug in 60 times)
    - Group 5--(have them do it 200 times)

   Record your outcomes. Ask students the following question: Do you think heads and tails have the same chance of coming up?

2. Teacher will generate a number cube or die (with the numbers 1-6) 10 times, 50 times, and a given amount from students. Ask students the following question: Do you think the outcomes are equally likely?

Motivational Activity 3:

Have students look at a picture of a six-sided die (on the computer) to answer question 1.

1. Which describes your chance of getting a 3?

   (a) 1 chance in 100 (b) 1 chance in 10
   (c) 1 chance in 6 (d) 1 chance in 2

For questions 2-5 look at the picture on the computer of the six-sided die again to write a number sentence that describes your chances:

2. What are your chances of getting the number 7?
3. What are your chances of getting a number less than 7?
4. What are your chances of spinning the number 1?
5. What are your chances of not spinning a 2?

6. Again, look at the number cube or die on the computer.
   Have each group describe events whose probabilities equal:

   a. 0

   b. $\frac{1}{6}$

   c. $\frac{2}{6}$

   d. $\frac{3}{6}$

   e. 1

LESSON 3

Goal: Develop an understanding of simple events

Objective:
The students will:

- Predict the probability of outcomes of simple experiments
- Test the predictions of simple experiments

Materials:

Marbles
Dice
Coins

Motivational Activity

1. There are 4 blue marbles, 6 white marbles, and 8 black marbles. First each group will predict an answer, then the teacher will try out the experiment on the computer:
   (a) How many marbles would you have to pick before you could be certain of having picked one matching pair? How does your guess compare to your results?
   (b) How many marbles would you have to pick to be certain to get one pair of black marbles? How does your guess compare to your results?

2. Suppose you have 5 red marbles, 4 yellow marbles, and 3 green marbles. Make predictions in your groups about what would happen for questions a-c, then discuss responses.

   (a) What is the probability of getting a yellow marble on the first draw?
   (b) What is the probability of getting a green marble on the first draw?
   (c) What is the probability of not getting a green marble on the first draw?
   (d) If you know that the probability of getting a yellow marble is 0, what does that tell you about the colors of the marbles in the bag?

3. One person from each group will come to the computer, generate a coin toss 25 times and again 50 times, and record the results after each toss.

   (a) From your experiment, what do you think is the probability of getting heads?
   (b) How do the two tosses compare to each other?
   (c) Do you think there is any change in the probability of the number of times heads will appear?

4. One student will generate a roll of a die on the computer 20 times. Each group will use this information to record their responses to questions a-e

   (a) Which number appeared the greatest number of times?
   (b) Which number appeared the least number of times?
   (c) Which numbers occurred about the same amount of times?
   (d) According to your experiment, which number seems most likely to occur?
   (e) Which number seems least likely to occur?
   (f) Try this experiment two more times; compare the three records you now have. Have you changed your opinion?

Lesson 1

manipulative group

Goal: Develop conceptual understanding of chance.

Objective:
The students will
- Explore concepts of chance through activities with spinners.
- Decide whether a game is fair or unfair

Materials:
Per cooperative learning group:
- Spinners
- paper and pencil to record information
- 1 activity board per group
- 1 color chip per team

Per Class:
- chart paper (transparency and overhead markers)

Procedures:

Students work in cooperative groups **(4-groups of four & 1-group of six) (2-spinners; 1-recorder; 1-reporter)**
Each group will divide into two teams for each of the three motivational activities

Motivational Introduction:
- Call on students to describe different games or activities they know are determined by chance or luck.
- Ask students what is meant by a fair game.
- After the activity have students discuss whether the spinners used were fair or unfair.

Motivational Activity 1:
- For this activity, use the spinners divided into red and gray sections
- Each of the 5-groups will divide into two teams (**team A & team B**).
- Players will start with the same spinner and the activity board.
- One player from each team will take turns spinning the spinner.
- One team will be gray and the other team will be red.
- Each time the spinner lands on the team color, that team gets to move one step towards the other team's goal. The team to reach the other teams goal is the winner.
- Students will play this game two more times using the two other spinners.
- Next, each team gets to choose a spinner to use for game 4(teams may choose the same one).
- Write down reason for choosing the spinner you did for game 4

- Reporters will share group results. The teacher will display all of the information for each group on the overhead.

Motivational Activity 2:

For games 1-4, each group will divide into two teams (A & B). Use the spinner divided into green and yellow sections. Each team will spin the spinner 4 times for each game. The team with the most points wins. Decide whether each game is fair or unfair. Each group should record their results.

Game 1:

Team A wins 1 point if the pointer stops on green. Team B wins 1 point if it stops on yellow.

Game 2:

Team A wins 1 point if the pointer stops on green. Team B wins 2 points if it stops on yellow.

Game 3:

Team A wins 2 points if the spinner stops on green. Team B wins 4 points if it stops on yellow.

Game 4:

Team A wins 4 points if the spinner stops on green. Team B wins 6 points if it stops on yellow.

Motivational Activity 3:

Use the spinner divided into orange, yellow, green, blue, purple, and red sections. For games 1-6, each group will divide into two teams (A & B). Each team will spin the spinner 3 times for each game. The team with the most points wins. Decide whether each game is fair or unfair. Each group should record their results.

Game 1:

Team A wins 1 point if the spinner lands on red, yellow, or blue. Team B wins 1 point if the spinner lands on orange, purple, or green.

Game 2:

Team A wins 1 point if the spinner lands on red or orange. Team B wins 1 point if the spinner lands on blue or green. No team gets any points if the spinner lands on purple or yellow.

Game 3:

Team A wins 3 points if the spinner lands on red, orange, or yellow. Team B wins 4 points if the spinner lands on purple, blue, or green.

Game 4:

Team A wins 5 points if the spinner lands on purple. Team B wins 1 point if the spinner lands on red, orange, yellow, green, or blue.

Game 5:

Team A wins 1 point if the spinner lands on blue and 2 points if the spinner lands on green. Team B wins 2 points if the spinner lands on red, orange, or yellow. No team gets any points if the spinner lands on purple.

Game 6:

Team A wins 1 point if the spinner lands on purple or blue and 2 points if the spinner lands on red. Team B wins 2 points if the spinner lands on orange or yellow and 1 point if the spinner lands on green.

LESSON 2

Goal: Develop an understanding of likely or unlikely events

Objective:
The Students will:

- Describe events as likely or unlikely
- Discuss the degree of likelihood using such words as certain, equally likely, and impossible.
- Find outcomes of probability experiments and decide if they are equally likely or not.
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

Materials:

Coins
Cups
Number cubes

Motivational Activity 1:
Each group will discuss the following terms with each other to check for understanding: certain, impossible, or likely(maybe)
Judge whether the following events are certain, impossible, or likely (maybe):

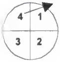
- It will rain tomorrow
- Drop a rock in water, and it will sink
- Trees will talk to us in the afternoon
- The sun will rise tomorrow morning
- Three students will be absent tomorrow
- George will go to bed before 8:30 tonight
- You will have two birthdays this year

Motivational Activity 2(work in groups)

1. Toss a coin 20 times (record your outcomes). Do you think heads and tails have the same chance of coming up?
2. Toss a number cube or die (with the numbers 1-6) 50 times (record your outcomes). Do you think the outcomes are equally likely?
3. Toss a paper cup 50 times (record the way it lands). The cup can land up ______, down, ______ or on its side ______. Do you think the outcomes are equally likely?

Motivational Activity 3:

Use the spinner to the right to answer question 1.



1. Which describes your chance of spinning a 3?

   (a) 1 chance in 100 (b) 1 chance in 10
   (c) 1 chance in 4 (d) 1 chance in 2

For questions 2-5 (use the spinner in 1) to write a number sentence that describes your chances.

2. What are your chances of spinning the number 5?
3. What are your chances of spinning a number less than 5?
4. What are your chances of spinning the number 3?
5. What are your chances of not spinning a 2?

6. Distribute a number cube (numbered 1-6) to each group.
   Have each group describe events whose probabilities equal:

   a. 0
   b. $\frac{1}{6}$
   c. $\frac{2}{6}$
   d. $\frac{3}{6}$
   e. 1

LESSON 3

Goal: Develop an understanding of simple events

Objective:
The students will:

- Predict the probability of outcomes of simple experiments
- Test the predictions of simple experiments

Materials:

Colored cubes
Spinner
Coins

Motivational Activity

1. In a drawer, there are 4 orange cubes, 6 white cubes, and 8 blue cubes. First each group will predict an answer, then each group will try out the experiment:
   (a) How many colored cubes would you have to pick before you could be certain of having picked one matching pair? How does your guess compare to your results?
   (b) How many colored cubes would you have to pick to be certain to get one pair of blue cubes? How does your guess compare to your results?

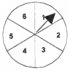
2. Suppose you have 5 red cubes, 4 yellow cubes, and 3 green cubes in a bag, make predictions in your group about what would happen for questions a-c, then discuss responses

   (a) What is the probability of getting a yellow cube on the first draw?
   (b) What is the probability of getting a green cube on the first draw?
   (c) What is the probability of not getting a green cube on the first draw?
   (d) If you know that the probability of getting a yellow cube is 0, what does that tell you about the colors of the cubes in the bag?

3. Each group will toss a coin 25 times and again 50 times, and record the results after each toss.

   (a) From your experiment, what do you think is the probability of getting heads?
   (b) How do the two tosses compare to each other?
   (c) Do you think there is any change in the probability of the number of times heads will appear?

Use the spinner below to answer questions 4-6.



4. If you spin a spinner like the one above, just one time, what are the chances of landing on 6?
5. What are the chances of landing on 3?
6. Each group will Spin a spinner (like the one above) 20 times. Each group will use this information to record their responses to questions a-e

   (a) Which number occurred the greatest number of times?
   (b) Which number occurred the least number of times?
   (c) Which numbers occurred about the same amount of times?
   (d) According to your experiment, which number seems most likely to occur?
   (e) Which number seems least likely to occur?
   (f) Try this experiment two more times; compare the three records you now have. Have you changed your opinion?

Lesson 1

manipulative/computer group

Goal: Develop conceptual understanding of chance.

Objective:
The students will

- Explore concepts of chance
- Decide whether a game is fair or unfair

Materials:
Per cooperative learning group:

- Spinners
- paper and pencil to record information
- dice (generated on the computer)
- 1 color chip per team

Per Class:

- chart paper (transparency and overhead markers)

Procedures:

Students work in cooperative groups **(4-groups of four & 1-group of six) (2-spinners; 1-recorder; 1-reporter)**
Each group will divide into two teams for each of the three motivational activities

Motivational Introduction:

- Call on students to describe different games or activities they know are determined by chance or luck.
- Ask students what is meant by a fair game.
- After the activities have students discuss, whether the spinners used were fair or unfair.

Motivational Activity 1 (***computer exercise***):

- For this activity, students will use a die generated from the computer software provided.
- Each of the 5-groups will divide into two teams (***team A & team B***).
- All the students on team A will be odd numbers (1,3, 5) and all the students on team B will be even numbers (2, 4, 6).
- The activity will start with one die generated on the computer.
- One person from each team (or the teacher) will take turns throwing the die on the computer.
- A team gets a point if any of their numbers come up on the die.
- The first team to get 15 points is the winner.
- This time the game will be played using two dice.
- One team will be even sum, and the other team will be odd sum.

- If the sum is odd, one team gets a point, if it's an even sum, the other team gets a point. The first team to get 10 points is the winner.

Motivational Activity 2 (**manipulative exercise**):

For games 1-4, each group will divide into two teams (A & B). Use the spinner divided into green and yellow sections. Each team will spin the spinner 4 times for each game. The team with the most points wins. Decide whether each game is fair or unfair. Each group should record their results.

Game 1:

Team A wins 1 point if the pointer stops on green. Team B wins 1 point if it stops on yellow.

Game 2:

Team A wins 1 point if the pointer stops on green. Team B wins 2 points if it stops on yellow.

Game 3:

Team A wins 2 points if the spinner stops on green. Team B wins 4 points if it stops on yellow.

Game 4:

Team A wins 4 points if the spinner stops on green. Team B wins 6 points if it stops on yellow.

Motivational Activity 3 (**computer exercise**):

Use a single die on the computer for this activity. For games 1-6, each group will divide into two teams *(A & B)*. Each team will throw one die 3 times for each game. The team with the most points wins. Decide whether each game is fair or unfair. Have students record their results.

Game 1:

Team A wins 1 point if the die lands on 2, 4, or 6. Team B wins 1 point if the die lands on 1, 3, or 5.

Game 2:

Team A wins 1 point if the die lands on 1 or 2. Team B wins 1 point if the die lands on 5 or 6. No team gets any points if the die lands on 3 or 4.

Game 3:

Team A wins 3 points if the die lands on 1, 2, or 4. Team B wins 4 points if the die lands on 3, 5, or 6.

Game 4:

Team A wins 5 points if the die lands on 3. Team B wins 1 point if the die lands on 1, 2, 4, 5, or 6.

Game 5:

Team A wins 1 point if the die lands on 6 and 2 points if the die lands on 5. Team B wins 2 points if the die lands on 1, 2, or 4. No team gets any points if the die lands on 3.

Game 6:

Team A wins 1 point if the die lands on 3 or 6 and 2 points if the die lands on 5. Team B wins 2 points if the die lands on 1 or 4 and 1 point if the die lands on 5.

LESSON 2

Goal: Develop an understanding of likely or unlikely events

Objective:
The Students will:

- Describe events as likely or unlikely
- Discuss the degree of likelihood using such words as certain, equally likely, and impossible.
- Find outcomes of probability experiments and decide if they are equally likely or not.
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

Materials:

Coins

Number cubes

Motivational Activity 1 **(manipulative/computer warm-up exercise):**
Each group will discuss the following terms with each other to check for understanding: certain, impossible, or likely (maybe)
Judge whether the following events are certain, impossible, or likely (maybe):

- It will rain tomorrow
- Drop a rock in water, and it will sink
- Trees will talk to us in the afternoon
- The sun will rise tomorrow morning
- Three students will be absent tomorrow
- George will go to bed before 8:30 tonight
- You will have two birthdays this year

Motivational Activity 2(students will work in their groups) **(computer exercise)**

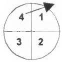
1. Have one student from each group come up and generate a coin toss on the computer
    - Group 1--(have them plug in 20 times)
    - Group 2--(have them plug in 25 times)
    - Group 3--(have them plug in 6 times)
    - Group 4--(have them plug in 60 times)
    - Group 5--(have them do it 200 times)

   Record your outcomes. Ask students the following question: Do you think heads and tails have the same chance of coming up?

2. Teacher will generate a number cube or die (with the numbers 1-6) 10 times, 50 times, and a given amount from students. Ask students the following question: Do you think the outcomes are equally likely?

Motivational Activity 3 **(manipulative exercise):**

Use the spinner to the right to answer question 1.



1. Which describes your chance of spinning a 3?

   (a) 1 chance in 100 (b) 1 chance in 10
   (c) 1 chance in 4 (d) 1 chance in 2

For questions 2-5 (use the spinner in 1) to write a number sentence that describes your chances.

2. What are your chances of spinning the number 5?
3. What are your chances of spinning a number less than 5?
4. What are your chances of spinning the number 3?
5. What are your chances of not spinning a 2?

6. Distribute a number cube (numbered 1-6) to each group.
   Have each group describe events whose probabilities equal:

   a. 0
   b. $\frac{1}{6}$
   c. $\frac{2}{6}$
   d. $\frac{3}{6}$
   e. 1

LESSON 3

Goal: Develop an understanding of simple events

Objective:
The students will:

- Predict the probability of outcomes of simple experiments
- Test the predictions of simple experiments

Materials:

Colored cubes
Spinner
Coins
Marbles
Dice

Motivational Activity **(computer exercise for #1-3)**

1. There are 4 blue marbles, 6 white marbles, and 8 black marbles. First each group will predict an answer, then the teacher will try out the experiment on the computer:
   (a) How many marbles would you have to pick before you could be certain of having picked one matching pair? How does your guess compare to your results?
   (b) How many marbles would you have to pick to be certain to get one pair of black marbles? How does your guess compare to your results?

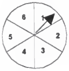
2. Suppose you have 5 red marbles, 4 yellow marbles, and 3 green marbles. Make predictions in your groups about what would happen for questions a-c, then discuss responses.

   (a) What is the probability of getting a yellow marble on the first draw?
   (b) What is the probability of getting a green marble on the first draw?
   (c) What is the probability of not getting a green marble on the first draw?
   (d) If you know that the probability of getting a yellow marble is 0, what does that tell you about the colors of the marbles in the bag?

3. One person from each group will come to the computer, generate a coin toss 25 times and again 50 times, and record the results after each toss.

   (a) From your experiment, what do you think is the probability of getting heads?
   (b) How do the two tosses compare to each other?
   (c) Do you think there is any change in the probability of the number of times heads will appear?

Use the spinner below to answer questions 4-6 **(manipulative exercise).**



4. If you spin a spinner like the one above, just one time, what are the chances of landing on 6?
5. What are the chances of landing on 3?
6. Each group will Spin a spinner (like the one above) 20 times. Each group will use this information to record their responses to questions a-e

   (a) Which number appeared the greatest number of times?
   (b) Which number appeared the least number of times?
   (c) Which numbers occurred about the same amount of times?
   (d) According to your experiment, which number seems most likely to occur?
   (e) Which number seems least likely to occur?
   (f) Try this experiment two more times; compare the three records you now have. Have you changed your opinion?

APPENDIX D

TEACHER CONSENT FORM

Teacher Consent Form

Thank you for agreeing to participate in my Ph.D. dissertation research project on probability. I am a graduate student in mathematics education at the University of Florida. The information gathered for this study will be used to investigate students' probability learning skills and concepts and how they may be affected by the use of concrete and computer-simulated manipulatives.

Your involvement in this research will consist of assisting in the administration of the pre and posttest and teaching the lessons on probability. I will prepare these lessons. You will provide scores for matching and comparison purposes from your semester grades for students participating in the study. All individual student scores will be kept confidential. Each student will be identified only by a number.

There is no risk involved for you. Participation or non-participation in this study will not affect you or your job. You have the right to withdraw permission for involvement without any penalty or prejudice. If you would like further information concerning this research, feel free to contact me at the University. Please sign the form on the space provided and return the bottom portion to me.

Thank you for your interest and support,

Felicia Taylor
Graduate Student
Mathematics Education
University of Florida
392-9191 Ext. 234

Return Bottom Portion

I have read and I understand the procedure described above. I agree to participate in Felicia Taylor's probability study. I have retained the top portion of this letter.

____ ____________

Teacher Signature Date

APPENDIX E
PARENTAL CONSENT FORM

College of Education
School of Teaching and Learning
PO Box117048
University of Florida
Gainesville, Fl 32611-7048

Parental Consent

Dear Parent/Guardian,

Your child's class has been selected to participate in my Ph.D. dissertation research project on probability. I am a graduate student in the School of Teaching and Learning at the University of Florida under the supervision of Dr. Thomasenia Lott Adams, conducting research on probability learning skills and concepts of elementary students. The purpose of this study is to compare student's probability learning skills and concepts under two different conditions, concrete and computer-simulated manipulatives. Manipulative materials are defined as objects or things that the student can feel, touch, handle, and move. The results of the study may help teachers better understand the amount of knowledge gained and allow them to design instructional practices accordingly. These results may not directly help your child today, but may benefit future students.

One-third of the participating children will participate in lessons using computer simulations for probability learning skills and concepts, while another group will use concrete manipulatives, and the third group will use a combination of computer simulations and concrete manipulatives. The researcher will develop the lessons. Children will be given a paper and pencil pretest before the start of the study and a posttest at the end of the study; but they will not have to answer any question they do not wish to answer. Your child's teacher will present the lessons and administer the tests during the Mathematics class period. The study will follow 3-4 lessons for probability. With your permission, the researcher will observe your child during the instructional period. Any written notes will only be accessible to the research team for verification purposes. At the end of the study, the notes will be discarded. Although the children will be asked to write their names on the test for matching purposes, their identity will be kept confidential to the extent provided by law. We will replace their names with code numbers. All individual student scores will be kept confidential. Results will only be reported in the form of group data. Participation or non-participation in this study will not affect the children's grades or placement in any programs. Students who do not wish to participate in the project will be given assignments for the class period in which the probability learning sessions will take place.

You and your child have the right to withdraw consent for your child's participation at any time without consequence. There are no known risks or immediate benefits to the participants. No compensation is offered for participation. Group results of this study will be available in December upon request. If you have any questions about this research project, please contact me or my faculty supervisor, Dr. Adams, at 392- 0761 ext. 243. Questions or concerns about research participants' rights may be directed to the

UFRIB office, University of Florida, Box 112250, Gainesville, Fl 32611-2250; (352) 392-0433. Please sign the form on the space provided indicating whether your child may participate, and have your child return the signed portion to his or her mathematics teacher. Also, inform your child whether he or she has permission to participate in this study.

Thank you for your interest and support,

Felicia Taylor
Graduate Student
Mathematics Education
University of Florida

I have read the procedure described above. I voluntarily give my consent for my child, ________________________________, to participate in Felicia Taylor's study of elementary students probability learning skills and concepts. I have received a copy of this description.

__________________________ _______________
Parent/Guardian Signature Date

__________________________ _______________
2nd Parent/Witness Signature Date

APPENDIX F
STUDENT CONSENT SCRIPT

Student Consent Script

My name is Felicia Taylor, and I am a student at the University. I would like to have you participate in 3-4 lessons using probability and then give you a posttest at the end of the lessons. You may stop at any time, and you will not have to answer any questions that you do not wish to answer. Would you like to do this?

REFERENCES

Allen, H. B. (1978). The use of cuisenaire rods to improve basic skills in seventh grade (Doctoral dissertation, Rutgers University, 1978). Dissertation Abstracts International, 39A, 2799.

Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1996). National Assessment of Educational Progress (NAEP) Technical Report. NCES Number 199945. Release date on the web, October 19, 2000. http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=1999452

Anderson, C. W. (1987). Strategic teaching in science. In B. F. Jones, A. S. Palinscar, D. S. Ogle, & E. G. Carr (Eds.), Strategic teaching and learning: Cognitive instruction in the content area (pp. 73-91). Alexandria, VA: Association for Supervision and Curriculum Development.

Anderson, T. (1996). What in the world is constructivism? Learning, 24(5), 48-51.

Anthony, G. (1996). Active learning in a constructivist framework. Educational Studies in Mathematics, 31(4), 349-369.

Babb, J. H. (1976). The effects of textbook instruction, manipulatives and imagery on recall of the basic multiplication facts (Doctoral dissertation, University of Southern Florida, 1975). Dissertation Abstracts International, 36A, 4378.

Battista, M. T. (1999). Fifth graders' enumeration of cubes in 3D arrays: Conceptual progress in an inquiry-based classroom. Journal for Research in Mathematics Education, 30(4), 417-448.

Battista, M. T., & Clements, D. H. (1996). Students' understanding of three-dimensional rectangular arrays of cubes. Journal for Research in Mathematics Education, 27, 258-292.

Battista, M., Clements, D. H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. Journal for Research in Mathematics Education, 29(5), 503-532.

Baxter, G., Shavelson, R., Herman, S., Brown, K., & Valadez, J. (1993). Mathematics performance assessment: Technical quality and diverse student impact. Journal for Research in Mathematics Education, 24(3), 190-216.

Bednarz, N., & Janvier, B. (1988). A constructivist approach to numeration in primary school: Results of a three year intervention with the same group of children. Educational Studies in Mathematics, 19, 299-333.

Bednarz, N., Janvier, B., Poirier, L., & Bacon, L. (1993). Socioconstructivist viewpoint on the use of symbolism in mathematics education. Alberta Journal of Educational Research, 39, 41-58.

Behr, M. (1976). Teaching experiment: The effect of manipulatives in second graders learning of mathematics, volume 1. PMDC Technical Report No. 11. (Eric Reproduction Service No. ED 144 809)

Berlin, D., & White, A. (1986). Computer simulations and the transition from concrete manipulation of objects to abstract thinking in elementary school mathematics. School Science and Mathematics, 86(6), 468-479.

Bishop, A. (1985). The social construction of amending: A significant development for mathematical education. For the Learning of Mathematics, 5(1), 24-28.

Bledsoe, J. C., Purser, J. D., & Frantz, N. R. (1974). Effects of manipulative activities on arithmetic achievement and retention. Psychological Reports, 35, 247-252.

Bolduc, E. J. (1970). A factorial study of the effects of three variables on the ability of the first-grade children to solve arithmetic addition problems (Doctoral dissertation, The University of Tennessee, 1969). Dissertation Abstracts International, 30A, 3358.

Bork, A. (1979). Interactive learning: Millikan lecture, American Association of Physics Teachers. American Journal of Physics, 47(1), 5-10.

Bring, C. R. (1972). Effects of varying concrete activities on the achievement of objectives in metric and non-metric geometry by students of grades five and six (Doctoral dissertation, University of Northern Colorado, 1971). Dissertation Abstracts International, 32A, 3775.

Broody, A., & Ginsburg, H. (1990). Children's mathematical learning: A cognitive view. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist view on the teaching and learning of mathematics (pp. 51-64). Reston, VA: National Council of Teachers of Mathematics.

Brooks, M. (1987). Curriculum development from a constructivist perspective. Educational Leadership, 44(4), 63-67.

Brown, C. K. (1973). A study of four approaches to teaching equivalent fractions to fourth-grade pupils (Doctoral dissertation, University of California, Los Angeles, 1972). Dissertation Abstracts International, 33A, 5465.

Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In W. Reeve (Ed.), The teaching of arithmetic. Tenth yearbook of the Council of Teaching of Mathematics (pp. 1-31). New York: Columbia University, Teachers College.

Bruner, J. (1960). The process of education. Cambridge, MA: Harvard University Press.

Bruner, J. (1986). Actual minds, possible worlds. Cambridge, MA: Harvard University Press.

Burns, M. (1996). How to make the most of math manipulatives. Instructor, 105(7), 45-51.

Burton, G. M. (1992). Young children's choices of manipulatives and strategies for solving whole number division problems. Focus on Learning Problems in Mathematics, 14(2), 2-17.

Caine, R., & Caine, G. (1991). Making connections: Teaching and the human brain. (ERIC Document Reproduction No. ED 335 141)

Callahan, J. J., & Jacobson, R. S. (1967). An experiment with retarded children and cuisenaire rods. Arithmetic Teacher, 14, 10-13.

Cambridge Conference on School Mathematics. (1963). Goals for school mathematics. Boston: Houghton Mifflin Company.

Campbell, P. F., & Stewart, E. L. (1993). Calculators and computers. In R. Jensen (Ed.), Research ideas for the classroom: Early childhood mathematics (pp. 251-268). New York: Macmillan.

Canny, M. E. (1984). The relationship of manipulative materials to achievement in three areas of fourth-grade mathematics: Computation, concept development and problem-solving (Doctoral dissertation, Virginia Polytechnic Institute and State University, 1983). Dissertation Abstract International, 45A, 775.

Carpenter, T. P. (1985). How children solve simple word problems. Education and Urban Society, 17(4), 417-425.

Carpenter, T. P., Moser, J., & Romberg, T. A. (1982). Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Erlbaum.

Chester, J., Davis, S., & Reglin, G. (1991). Math manipulative use and math achievement of third-grade students. (Eric Reproduction Service No. ED 339 591)

Choi, B., & Gennaro, E. (1987). The effectiveness of using computer simulated experiments on junior high students' understanding of the volume displacement concept. Journal for Research in Science Teaching, 26(6), 539-552.

Clement, J. (1991). Constructivism in the classroom. A review of L. P. Steffe & T. Wood (Eds.), Transforming children's mathematical education: International perspectives. Journal for Research in Mathematics Education, 22, 422-428.

Clements, D. H. (1998) Young children and technology. (ERIC Document Reproduction Service No. ED 416 991)

Clements, D., & Battista, M. (1990). Constructivist learning and teaching. Arithmetic Teacher, 38, 34-35.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 420-464).

Clements, D. H., Crown, W. D., Kantowski, M. G. (1991). Primary graphing and probability workshop. Glenview, IL: Scott, Foresman.

Clements, D., & McMillan, S. (1996). Rethinking "concrete" manipulatives. Teaching Children Mathematics, 2(5), 270-279.

Clements, D. H., Swaminathan, S., Hannibal, M., & Sarama, J. (1999). Young children's concepts of shape. Journal for Research in Mathematics Education, 30(2), 192-212.

Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. Educational Psychologist, 23, 87-103.

Cobb, P. (1995). Cultural tools and mathematical learning: A case study. Journal for Research in Mathematics Education, 28(4), 362-385.

Cobb, P., & Steffe, P. (1983). The constructivist researcher as teacher and model builder. Journal for Research in Mathematics Education 14(2), 83-94.

Cobb, P., Wood, T., & Yackel, E. (1990). Classrooms as learning environments for teachers and researchers. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics. Monograph #4 (pp. 125-146). Reston, VA: NCTM.

Cobb, P., Wood, T., & Yackel, E. (1991). Learning through problem solving: A constructivist approach to second grade mathematics. In E. von Glasersfeld (Ed.), Radical constructivism in mathematics education (pp. 157-176). Kluwer: Dordrect, Netherlands.

Cobb, P., Wood, T., & Yackel, E. (1992). Interaction and learning in mathematics classroom situations. Educational Studies in Mathematics, 23(1), 99-122.

Cobb, P., Wood, T., Yackel, E., Nicholls, J., Wheatley, G., Trigatti, B., & Perlwitz, M. (1991). Assessment of a problem-centered second-grade mathematics project. Journal for Research in Mathematics Education, 22, 3-29.

Confrey, J. (1990). What constructivism implies for teaching. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics (pp. 107-122). Reston, VA: National Council of Teachers of Mathematics.

Confrey, J. (1991). The use of contextual problems and multi-representational software to teach the concept of functions. New York: Apple Computer and National Science Foundation.

Connell, M. (1998). Technology in constructivist mathematics classrooms. Journal of Computers in Mathematics and Science Teaching, 17(4), 311-338.

Cuisenaire. (1992). Learning with Cuisenaire rods. White Plains, NY: Cuisenaire Company of America, Inc.

Davidson, N. (1990). Cooperative learning in mathematics: A handbook for teachers. Menlo Park, CA: Addison Wesley.

Davis, R. B., Maher, C. A., & Noddings, N. (1990). Suggestions for the improvement of mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist view on the teaching and learning of mathematics (pp. 187-191). Reston, VA: National Council of Teachers of Mathematics.

Derderian, D. L. (1980). The effects of teaching mathematics through movement activities to sixth graders (Doctoral dissertation, Boston University School of Education, 1980). Dissertation Abstracts International, 41A, 2005.

Dewey, J. (1963). The nature of subject matter. In R. R. Archamlaull (Ed.), John Dewey on education (pp. 359-372). Chicago: University of Chicago Press.

Drier, H. (2000). Children's probabilistic reasoning with a computer microworld (Doctoral dissertation, University of Virginia, 2000). Dissertation Abstracts International, 61-06A, 2224.

Earhart, E. (1964). Evaluating certain aspects of a new approach to mathematics in the primary grades. School Science and Mathematics, 64, 715-720.

Ekman, L. G. (1967). A comparison of the effectiveness of different approaches to the teaching of addition and subtraction algorithms in the third grade (Doctoral dissertation, University of Minnesota, 1966). Dissertation Abstracts International, 27, 2273.

Fast, G. R. (1999). Analogies and reconstruction of probability knowledge. School Science and Mathematics, 99(5), 230-240.

Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. American Education Research Journal, 30, 555-583.

Fishbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? Educational Studies in Mathematics, 15, 1-24.

Fischbein, M., Nello M., & Marino, M. (1991). Factors affecting probabilistic judgements in children and adolescents. Educational Studies in Mathematics, 22(6), 523-549.

Franke, M., & Carey, D. A. (1997). Young children's perceptions of mathematics in problem-solving environments. Journal for Research in Mathematics Education, 28(1), 8-25.

Fuson, K. (1975). The effects on preservice elementary teachers of learning mathematics and means of teaching mathematics through the active manipulation of materials. Journal for Research in Mathematics Education, 6(1), 51-63.

Gardner, C. M., Simons, P. E., & Simpson, P. D. (1992). The effects of CAI and hands-on activities on elementary students attitudes and weather knowledge. School Science and Mathematics, 92(6), 334-336.

Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. Journal for Research in Mathematics Education, 19(1), 44-63.

Ginsburg, H. A. (1983). The development of mathematical thinking. New York: Academic Press.

Glatthorn, A. A. (1994). Constructivism: Implications for curriculum. International Journal of Educational Reform, 3(4), 449-455.

Greening, T. (1998). Building the constructivist toolbox: An exploration of cognitive technologies. Educational Technology, 38(2), 23-35.

Harris, M. (1993). The effect of manipulatives on developing mathematics achievement and attitudes of seventh-grade students (Doctoral dissertation, Memphis State University, 1993). Dissertation Abstract International, 54A, 3664.

Harvey, F. A., & Charnitski, C. W. (1998). Improving mathematics instruction using technology: A Vygotskian perspective. (Eric Document Reproduction Service No. ED 423 837)

Hausfathers, S. J. (1996). Vygotsky and schooling: Creating a social context for learning. Action in Teacher Education, 18(2), 1-10.

Hinzman, K. P. (1997). Use of manipulatives in mathematics at the middle school level and their efforts on students' grades and attitudes. (Eric Reproduction Service No. ED 411 150)

Hoffmann, R. I. (1988). Educational technology for elementary and middle schools. Journal of Educational Technology Systems, 16(4), 299-314.

Horak, V. M., & Horak, W. J. (1983). Let's do it: Take a chance. Arithmetic Teacher, 30(9), 8-13.

Huff, D. (1959). How to take a chance. New York: W.W. Norton.

Hunting, R. (1983). Alan: A case study of knowledge of units and performance with fractions. Journal for Research in Mathematics Education, 14, 182-197.

Jaworski, B. (1992). Mathematics teaching: What is it? For the Learning of Mathematics, 12(1), 8-14.

Jiang, Z. (1993). Students' learning of introductory probability in a mathematical microworld (Doctoral dissertation, University of Georgia, 1992). Dissertation Abstract International, 54-09A, 3360.

Jiang, Z., & Potter, W. (1994). A computer microworld to introduce students to probability. Journal of Computers in Mathematics and Science Teaching, 13(2), 197-222.

Johnson, R. L. (1971). Effects of varying concrete activities on achievement of objectives in perimeter, area, and volume by students of grades four, five, and six (Doctoral dissertation, University of Chicago, 1970). Dissertation Abstracts International, 31A, 4624.

Jones, G., Langrall, C., Thornton, C., & Mogill, A. (1999). Students' probabilistic thinking in instruction. Journal for Research in Mathematics Education, 30(5), 487-521.

Kamii, C. (1985). Young children reinvent arithmetic: Implication of Piaget's theory. Columbia, NY: Teacher College Press.

Kamii, C., & Ewing, J. K. (1996). Basing teaching on Piaget's constructivism. Childhood Education, 72(5), 260-264.

Kanemoto, A. E. (1998). Effects of math manipulatives on students learning place value (Master thesis, California State University, Long Beach, 1998). Master Abstract International, 36-05, 1227.

Kapadia, R., & Borovcnik, M. (1991). The educational perspective. In R. Kapadia & M. Borovcnik (Eds.), Chance encounters: Probability in education (pp. 1-26). Dordrecht, Boston: Kluwer Academic Publishers.

Kaput, J. J. (1991). Notations and representations as mediators of constructive processes. In E. von Glaserfeld (Ed.), Constructivism in mathematics education (pp. 53-74). Dordrecht, Netherlands: Kluwer.

Kennedy, L. M., & Tipps, S. (1994). Guiding children's learning of mathematics. Belmont, CA: Wadsworth.

Kenold, C., Pollatsek, A., Lohmeier, J., & Lipson, D. (1993). Inconsistencies in students' reasoning about probability. Journal for Research in Mathematics Education, 24, 392-414.

Kim, S. (1993). The relative effectiveness on hands-on and computer-simulated manipulatives in teaching seriation, classification, geometrics, and arithmetic concepts to kindergarten children (Doctoral dissertation, University of Oregon, 1993). Dissertation Abstract International, 54-09A, 3319.

Kirkwood, J. (1992). Elementary school math and technology education. Technology Teacher, 33(3), 29-31.

Koehler, M., & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 115-126). New York: Macmillan.

Labinowicz, E. (1985). Learning from students: New beginnings for teaching numerical thinking. Menlo Park, CA: Addison-Wesley.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. Cognition and Instruction, 3, 305-342.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. American Educational Research Journal, 27, 29-63.

Langbort, C. R. (1988). Jay lids-an unusual math manipulative. Arithmetic Teacher, 36(3), 22-25.

Leiken, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. Journal for Research in Mathematics Education, 28(3), 331-354.

Lesh, R. (1979). Some trends in research and the acquisition and use of space and geometry concepts. Paper from the 2nd International Conference for the Psychology of Mathematics Education. Institute for Didactics in Mathematics, Bielefeld, Germany.

Lesh, R. (1990). Computer-based assessment of higher order understanding and processes in elementary mathematics. In G. Kulm (Ed.), Assessing higher order thinking in mathematics (pp. 81-110). Washington, DC: AAAS.

Letteri, F. (1980). Effects of the use of attribute materials on first-grade and second-grade children's development of the concept of number (Doctoral dissertation, Temple University, 1980). Dissertation Abstracts International, 41A, 1982.

Lunetta, V. N., & Hofstein, A. (1981). Simulations in science education. Science Education, 65(3), 243-252.

Mack, N. K. (1990). Learning fractions with understanding: Building on informal knowledge. Journal for Research in Mathematics Education, 21, 16-32.

Mack, N. K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. Journal for Research in Mathematics Education, 26(5), 422-441.

Maher, C. (1991). Implementing a thinking curriculum in mathematics. Journal of Mathematical Behavior, 10(3), 219-224.

Marzola, E. (1987). Using manipulative in math instruction. Journal of Reading, Writing, and Learning Disabilities International, 3(1), 9-20.

Mathematical Sciences Education Board. (1990). Reshaping school mathematics: A philosophy and framework for curriculum. Washington, DC: National Academy Press.

McClung, L. W. (1998). A study on the use of manipulatives and their effect on student achievement in a high school algebra 1 class. (Eric Reproduction Service No. ED 425 077)

Midkiff, R., & Thomasson, R. D. (1993). A practical approach to using learning styles in math instruction. (ERIC Document Reproduct Service No. ED 367 542)

Milton, K. C. (1975). Probability in the primary school. Australian Mathematics Teacher, 31(5), 169-178.

Minstrell, J. A. (1989). Teaching science for understanding. In L. Resnick & L. E. Klopfer (Eds.), Toward the thinking curriculum: Current cognitive research, 1989 ASCD yearbook (pp. 129-149). Alexandria, VA: Association for Supervision and Curriculum Development.

Moore, D. S. (1990). Uncertainty. In L. A. Steen (Ed.), On the shoulders of giants: New approaches to numeracy (pp. 95-117). Washington, DC: National Academy Press.

National Council for Supervisor of Mathematics (NCSM). (1989). Essential mathematics for the twenty-first century: The position of the National Council of Supervisors of Mathematics. Arithmetic Teacher, 31(1), 44-46.

National Council of Teachers of Mathematics (NCTM). (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM). (1998). An NCTM position statement on: The use of technology in the learning and teaching of mathematics [On-line, May 18, 2001]. Available: http://www.nctm.org/about/position statements/position statement13.htm.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

Newman, C. M., Obremski, T., & Schaeffer, R. L. (1987). Exploring probability. Palo Alto, CA: Dale Seymour Publications.

Nicholls, J. G., Cobb, P., Wood, T., Yackell, E., & Patashnick, M. (1990). Assessing students' theories of success in mathematics: Individual and classroom differences. Journal for Research in Mathematics Education, 21(2), 109-122.

Nichols, E. J. (1972). A comparison of two methods of instruction in multiplication and division for third-grade pupils (Doctoral dissertation, University of California, Los Angeles, 1971). Dissertation Abstracts International, 32A, 6011.

Nickel, A. P. (1971). A multi-experience approach to conceptualization for the purpose of improvement of verbal problem-solving in arithmetic (Doctoral dissertation, University of Oregon, 1971). Dissertation Abstracts International, 32A, 2917-2918.

Noddings, N. (1990). Constructivism in mathematics education. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics (pp. 7-18). Reston, VA: NCTM.

Novak, J. D., & Gowin, D. B. (1984). Learning how to learn. Cambridge, MA: Cambridge University Press.

Ohanian, S. (1992). Energize your math program. Instructor, 101(8), 44-46.

Owen, L., & Lamb, C. (1996). Bringing the NCTM standards to life: Best practices from elementary educators. Princeton, NJ: Eye on Education.

Papert, S. (1980). Mindstorms: Children, computers, and powerful ideas. Basic Books: New York.

Perl, T. (1990). Manipulatives and the computer: A powerful partnership for learners. Classroom Computer Learning, 10(6), 20-22, 26-29.

Piaget, J. (1970). Structuralism. New York: Basic Books. (Eric Document Reproduction Service No. ED 059 072)

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? Educational Studies in Mathematics, 26, 165-190.

Pugalee, D. K. (1999). Rolling the dice: Developing an understanding of experimental and theoretical probability. Learning and Leading with Technology, 26(6), 18-21.

Purser, J. D. (1974). The relation of manipulative activities, achievement, and retention in a seventh-grade mathematics class: An exploratory study (Doctoral dissertation, University of Georgia, 1973). Dissertation Abstracts International, 34A, 6954-6955.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), Advances in instructional psychology (pp. 41-95). Hillsdale, NJ: Erlbaum.

Reys, R. E. (1971). Considerations for teacher using manipulative materials. Arithmetic Teacher, 18(8), 551-558.

Riley, R. W. (1998). The state of mathematics education: Building a strong foundation for the 21st century. Conference of American Mathematical Society and Mathematical Association of America [On-line, October 18, 2000]. Available: www.ed.gov/speeches/01-1998/980108.html.

Robinson, E. B. (1978). The effects of a concrete manipulative on attitude toward mathematics and levels of achievement and retention of a mathematical concept among elementary students (Doctoral dissertation, East Texas State University, 1978). Dissertation Abstracts International, 39B, 1335.

Romero, R. C. (1979). Student achievement in a pilot cureton reading, cuisenaire mathematics program, and bilingual program of an elementary school (Doctoral dissertation, Northern Arizona University, 1977). Dissertation Abstracts International, 39A, 4033.

Rubink, W. L., & Taube, S. R. (1999). Mathematical connections from biiology: "Killer" bees come to life in the classroom. Mathematics Teaching in the Middle School, 4 (6), 350-356.

Saenz-Ludlow, A., & Walgamuth, C. (1998). Third graders' interpretations of equality and the equal symbol. Educational Studies in Mathematics, 35(2), 153-187.

Schaeffer, R. I. (1984). The role of statistics in reviatlizing precollege mathematics and science education. In proceedings of the section on statistical education. Washington, DC: American Statistical Association.

Scheer, J. K. (1985). Manipulatives make math meaningful for middle schoolers. Childhood Education, 62(2), 115-121.

Schoenfeld, A. H. (1987). Cognitive science and mathematics education. Hillsdale, NJ: Erlbaum.

Schulte, P. L. (1996). A definition of constructivism. Science Scope, 20(3), 25-27.

Schunk, D. H. (1991). Learning theories: An educational perspective. New York: Merrill.

Shaughnessy, M. J. (1992). Research in probability and statistics: Reflections and directions. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 465-494). New York, NY: Macmillan.

Shaughnessy, M. J. (1993). Connecting research to teaching: Probability and statistics. Mathematics Teacher, 86(3), 244-248.

Siegel, M. Borasi, R., & Fonzi, J. (1998). Supporting students' mathematical inquiries through reading. Journal for Research in Mathematics Education, 29(4), 378-413.

Simon, M. A. (1991). Report of classroom observations: Evaluation of the mathematics leadership network. Unpublished manuscript available from the author.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics Education, 26, 114-145.

Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. Journal for Research in Mathematics Education, 201(5), 498-505.

Steffe, L., & Gale, J. (Eds.),(1995). Constructivism in education. Hillsdale, NJ: Lawrence Erlbaum.

Stockdale, L. N. (1980). The effects of manipulative on student performance on multiplication and factorization of second-degree polynomials (Doctoral dissertation, Kent State University, 1980). Dissertation Abstracts International, 41A, 1931.

Stone, J. I. (1987). Early childhood math: Make it manipulative. Young Children, 42(6), 16-23.

Streefland, L. (1991). Fractions in realistic mathematics education. Norwell, MA: Kluwer Academic Publishers.

Strommen, E. F., & Lincoln, B. (1992). Constructivism, technology, and the future of classroom learning. Education and Urban Society, 24(4), 466-476.

Suydam, M. N. (1984). Research report: Manipulative materials. Arithmetic Teacher, 37(5), 27.

Swift, J. (1982). The first international conference on teaching statistics. Statistics Teacher Network, 1-2.

Tambanjong, A. (1983). A comparison of the effectiveness of using and not using manipulative materials in teaching addition and subtraction to first grade students in Bangkok, Thailand (Doctoral dissertation, University of Houston, 1983). Dissertation Abstract International, 44A, 1365.

Taylor, L., & Brooks, K. (1986). Building math confidence by overcoming math anxiety. From theory to practice. Adult Literacy and Basic Education, 10(1), insert.

Terry, M. (1995). An investigation of differences in cognition when utilizing math manipulatives and math manipulative software (Doctoral dissertation, University of Missouri-Saint Louis, 1995). Dissertation Abstract International, 56-07A, 2650.

Thompson, P. (1992). Notations, conventions, and constraints: Contributions to effective uses of concrete materials in elementary mathematics. Journal for Research in Mathematics Education, 23(2), 123-147.

Thornton, C., & Wilson, S. (1993). Classroom organization and models of instruction. In R. J. Jenson (Ed.), Research ideas for the classroom: Early childhood mathematics (pp. 269-293). New York: Macmillan.

Tobin, A. (1974). An experimental study of teaching mathematics to retarded educable children in elementary school through the use of concrete materials in an activity-centered environment (Doctoral dissertation, Temple University, 1974). Dissertation Abstracts International, 35A, 3412.

Toney, J. S. (1968). The effectiveness of individual manipulation of instructional material as compared to a teacher demonstration in developing understanding in mathematics (Doctoral dissertation, Indiana University, 1968). Dissertation Abstracts International, 33A, 4253-4254.

Trueblood, C. R. (1968). A comparison of two techniques for using visual-tactual devices to teach exponents and non-decimal bases in elementary school mathematics (Doctoral dissertation, Pennsylvania State University, 1967). Dissertation Abstracts International, 29A, 1831-1832.

Tsai, C. (1998). An analysis of scientific epistemological beliefs and learning orientations of Taiwanese eighth graders. Science Education, 82(4), 473-489.

Tzur, R. (1999). An integrated study of children's construction of improper fractions and the teacher's role in promoting that learning. Journal for Research in Mathematics Education, 30(4), 390-416.

Vahey, P. H. (1998). Promoting student understanding of elementary probability using a technology mediated inquiry environment (Doctoral dissertation, University of California, Berkeley, 1996). Dissertation Abstract International, 60-03A, 0712.

Van de Walle, J. A. (1998). Elementary and middle school mathematics: Teaching developmentally (3rd ed.). New York: Longman.

von Glassersfeld, E. (Ed.), (1991). Radical constructivism in mathematics education. Dordrecht, Netherlands: Kluwer.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge MA: Cambridge University Press.

Wallace, P. (1974). An investigation of the relative effects of teaching a mathematical concept via multisensory models in elementary school mathematics (Doctoral dissertation, Michigan State University, 1974). Dissertation Abstracts International, 35B, 2989.

Watson, J. M., Collis, K. F., & Moritz, J. B. (1997). The development of chance measurement. Mathematics Education Research Journal, 9(1), 60-82.

Weibe, J. H. (1988). Teaching elementary mathematics in a technological age. (Eric Document Reproduction Service No. ED 297 941)

Wheatley, G. (1979). Calculators in the classroom: A proposal for curricular change. (Eric Document Reproduction Service No. ED 175 631)

Wiegel, H. G. (1998). Kindergarten students' organization of counting in joint counting tasks and the emergence of cooperation. Journal for Research in Mathematics Education, 29(2), 202-221.

Willoughby, S. S. (1990). Mathematics education for a changing world. Alexandria, VA: ASCD Publications.

Wirtz, R. W. (1977). Making friends with numbers. Washington, DC: Curriculum Development Associates.

Wood, T., & Sellers, P. (1996). Assessment of a problem-centered mathematics program: Third grade. Journal for Research in Mathematics Education, 27(2), 337-353.

Yackel, E., Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In T. Cooney, & C. R. Hirsch (Eds.), Teaching and learning mathematics in 1990's, 1990 Yearbook of the National Council of Teachers of Mathematics (pp. 12-21). Reston, VA: NCTM.

Yeatts, K, (1991). Manipulatives: Motivating mathematics. (Eric Document Reproduction Service No. ED 355 097)

Zazkis, R., & Campbell, S. (1996). Divisibility and multiplicative structure of natural numbers: Preservice teachers' understanding. Journal for Research in Mathematics Education, 27(5), 540-563.

BIOGRAPHICAL SKETCH

Felicia Mae Taylor was born on September 2, 1965, in Miami, Florida. She graduated from public high school in 1983. In 1988, she graduated from the University of Florida with a Bachelor of Science degree in mathematics. From 1988 to 1997, she taught high school mathematics in Alachua, Florida. Upon receiving the McKnight Doctoral Fellowship, she left to begin her doctoral studies in mathematics education at the University of Florida in Gainesville, Florida. In 1995, she married Reginald Eugene Taylor. She completed a Master of Arts degree in mathematics education from University of Florida in 1998. In 2001, she completed the doctorate in instruction and curriculum from the University of Florida. She joined the faculty of the Department of Urban Education of the University of Houston-Downtown in 2000.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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